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Are Long Waves 50 Years? Reexamining Economic and Financial Long Wave Periodicities in Kondratieff and Schumpeter

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ABSTRACT

In 1925, the Russian economist, Nikolai D. Kondratieff first presented his identification and analysis of a 'long wave' cycle of approximately 50 years in twenty-five economic and financial time series across the major capitalist economies. The statistical evidence for their existence was based on nine-year centered moving averages of residuals from econometric time-trend models for eight English and five French time series that spanned the 18th, 19th and early 20th centuries. Schumpeter supported and promoted Kondratieff's estimate of a long wave periodicity which was consistent with his trigonometric models published in *Business Cycles* (1939). While Schumpeter never attempted to measure the statistical association between his theoretical values and historical observations, he identified long waves in the numerous graphs and charts in *Business Cycles*. Kondratieff's original data is used to estimate long wave periodicities by replicating his published models and smoothed residuals to verify specific years of turning points. 'Unobserved component models' are also used to extract long wave periodicities from Kondratieff's data as well as from new long-term time series recently published by the Bank of England. The new estimates confirm an endogenously-propagated long cycle of about fifty ears.

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> Analyzing business cycles means neither more nor less than analyzing the economic process of the capitalist era. … Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them. (Schumpeter 1939, p. v)

1. Introduction

Understanding and measuring the long-term cyclical movements of capitalist economies were central objectives in the works of Kondratieff (1926 [1998]) and Schumpeter (1939).[1] While Kondratieff coined the term 'economic long waves,' Schumpeter made

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[1]The non-abridged German translation 'Die lamgen Wellen der Konjunktur' was published shortly after the Russian original in the Archiv für Sozialwissenschaft und Sozialpolitik (ASS); Schumpeter and Alfred Weber (younger brother of Max Weber) were Associate Editors. Beginning in 1926, Schumpeter facilitated the translation into English (by his student Wolfgang Stopler); Kondratieff's article was subsequently published in the *Review of Economic Statistics* in 1935.

numerous references to the phenomena which he described and identified in charts and graphs in *Business Cycles*.[2] Kondratieff estimated the average long cycle to be about 50 years based on specific years of peaks and troughs for twenty-five time series from 1780 through 1925. Graphs of nine-year centered moving averages of residuals from econometric time-trend models were used to identify long cycle peaks and troughs. In contrast, Schumpeter never applied statistical techniques to estimate long cycle periodicities. Instead, he specified three models based on trigonometric sine functions to generate a short, intermediate, and long cycle with periodicities of about 3, 9, and 57 years, respectively.[3] Unfortunately, Schumpeter never attempted to measure the correlation between an actual time series and simulated values generated by his trigonometric equations. In his review of *Business Cycles*, Kuznets (1940) admonished Schumpeter for the lack of a 'statistical serviceable procedure' to validate his 'rather rigid' mathematical model of business cycles. Nevertheless, Schumpeter heartily endorsed and promoted Kondratieff's estimate of a 50-year economic and financial long cycle.

Historians of econometric thought have largely ignored Schumpeter's ingenious application of trigonometry to model business cycle periodicities which anticipated the incorporation of trigonometric functions into 'unobserved components models' (UCMs) developed by Nerlove, Grether, and Carvalho (1995) and Koopman et al. (2009).[4] Schumpeter (1954, p. 742) traced the history of economic thought regarding the shape and frequency of business cycles, highlighting important early contributions by Tooke and Overstone who 'spoke of a "periodicity" of these cycles' characterized as a 'definite sequence of phases irrespective of duration.' In this same passage, Schumpeter was critical of the popularity among 19th-century classical economists of a 'ten-year cycle'[5] which 'even Marx experimented with in a noncommittal manner' unlike Jevons and Juglar whom he credits with having made 'seminal performances' to the understanding of business cycles.

Two main ideas about long wave cycles and their periodicities unite Kondratieff and Schumpeter. First, any economic or financial time series can be decomposed into a latent (unobservable) trend plus one or more cyclical patterns. Both economists cited Wesley Clair Mitchell's (1927) canonical work on time series decomposition which applied a time-trend and moving averages to identify the trend, cycle, seasonal, and irregular components of any time series.[6] The second unifying theme is that business cycle lengths

[2]Kondratieff's term 'long cycles' was translated into German as 'long waves' which may be related to the fact that Schumpeter described 'wavelike movements' in the title of a seminar he gave at Harvard in January 1914. Although Kondratieff preferred the metaphor of an economic or financial 'wave', the concept of a long wave became synonymous with Kondratieff cycles.

[3]Schumpeter's four cycles depicted on p. 213 and defined (in degrees) on p. 1051 as: $\alpha = \sin(360/684)(t)$ — a short cycle of 33 months or 3 1/2 years; $\beta = 3\sin(360/114)(t)$ — an intermediate cycle of 114 months or 9 1/2 years; $\gamma = \sin(360/38)(t)$ — a long cycle of 684 months or 57 1/2 years. The fourth curve is the sum of $\alpha + \beta + \gamma$.

[4]Nerlove, Grether, and Carvalho (1995) provided a comprehensive summary of the origins of UCMs in chapter 1, entitled 'A History of the Idea of Unobserved Components in the Analysis of Economic Time Series.' Although Kondratieff, Schumpeter and Fellner (1956) are mentioned as progenitors, on p. 1 they note that the: 'The literature dealing with the existence of long cycles is not treated in any detail.'

[5]Marx (1867 [1967], p. 637) 'For modern industry with its decennial cycles and periodic phases, which moreover, as accumulation advances, are complicated by irregular oscillations following each other more and more quickly, that would indeed be a beautiful law, which pretends to make capital dependent on the absolute variation of the population, instead of regulating the demand and supply of labour by the alternate expansion and contraction of capital, the labour market now appearing relatively under-full, because capital is expanding, now again over-full because it is contracting.'

[6]See Appendix 1 for a description of the classical time series decomposition methodology.

reflect different but interrelated economic and financial factors which are fundamentally *endogenous* to the capitalist system. Kondratieff was quite explicit about the last theme and criticized claims that long waves were driven by stochastic/exogenous forces such as wars/revolutions, new technologies, the opening up of new territories and new sources of gold production.[7] In contrast, Kondratieff (1935, p. 115) suggested that each of these factors was dependent upon the broader cumulative impact of economic 'possibilities' and 'circumstances' which propel the internal dynamics of the capitalist system:

> In asserting the existence of long waves and in denying that they arise out of random causes, we are also of the opinion that the long waves arise out of causes which are *inherent* in the *essence* of the capitalistic economy. (Emphasis added)

In this way, Kondratieff's theory of the long wave is connected to Smith, Ricardo, and Marx's notion that capital accumulation — in both its physical and financial forms — is a slow-moving endogenous process that persists over several decades in a positive, and then a negative direction. The secular accretion of capital initially causes an improvement in productive forces, capacities and material conditions, which is then followed by their respective diminution and/or decline. The timing and breadth of Kondratieff's argument is supported by the year he identified as a long cycle peak or trough in his 'Table I' (1935, p. 110) which included prices, wages, various measures of industrial output, imports, exports, bank deposits, and interest rates across France, England, the US, Germany, and the 'whole world.'[8] Kondratieff (1935, p. 105) estimated that the average length of a long wave was about 50 years with shorter 'business' or 'intermediate cycles' of 7–11 years and still 'shorter waves of about three and one-half years has recently been shown to be probable.'[9]

Schumpeter (1939, p. 164) often cited Kondratieff when identifying and labeling a long wave:

> It was N.D. Kondratieff who brought the phenomena before the scientific community and who systematically analyzed all the material available to him on the assumption of the presence of a Long Wave, characteristic of the capitalist process.[10]

Although Schumpeter's *Business Cycles* (1939) are permeated by Kondratieff's long cycles, they did not play any role in his *The Theory of Economic Development* (1934).[11] On the other hand, he recognized that the process of capitalist dynamics where 'progress unstabilizes the economic world' is not likely to be theoretically or politically palatable to mainstream theorists.[12] Schumpeter labored to reconcile his theory of economic growth with the non-evolutionary world of Walrasian perfect competition and static

[7]Schumpeter considered basic technological changes or 'General Purpose Technologies' as the fundamental driver of long waves. In contrast, Kondratieff did not identify any single determinant of long wave cycles, however, as Freeman and Louca (2001, p. 109) observe, 'One of the most relevant examples was that of Kondratiev, to whom endogeneity meant that all the relevant variables could be defined as having been generated by the economic system itself.'

[8]In the most complete translation of the original paper, Kondratieff (1926 [1998]) identified turning point years for three long cycles across 25 variables (p. 37) and estimated econometric models for all 8 English variables and 5 of 10 French variables; no models were published for the 4 USA variables, 1 German variable and 2 world variables.

[9]The short cycle is credited to Kitchin (1923) whom both Kondratieff and Schumpeter referenced.

[10]In the footnote to this sentence, Schumpeter notes that Mitchell acknowledged the 'existence' of long waves but did not ascribe any importance other than calling them 'merely empirical.'

[11]The 1934 edition is based on the revised 1926 German edition, which was finalized before the publication of Kondratieff (1926).

[12]Ironically, Kondratieff's emphasis on endogenous factors to explain capitalist long cycles likely contributed to his disaffection with Soviet contemporaries (e.g., Trotsky), resulting in his imprisonment in Suzdal political prison and eventual execution on 17 September 1938.

macroeconomic equilibrium. Thus, Schumpeter needed to demonstrate how innovation is the critical endogenous determinant of business cycles which propels the economy from one equilibrium position to another:

> It is, after all, only common sense to realize that, but for the fact that economic life is in a process of incessant *internal* change, the business cycle as we know it would not exist. (p. 164)

However, Schumpeter (p. 139) also acknowledged 'the fact that while innovation would suffice to produce alternating prosperities and depressions, of course, that these cycles are actually … business cycles.' Since innovation is often uneven in its development, diffusion, and impact, Schumpeter (1927, 1934) allowed for the possibility that exogenous factors such as natural disasters, seasonal/sunspot activity on agricultural output, political upheavals (e.g., war, revolutions) could shape the frequency and amplitude of individual business cycles. While Schumpeter expected these factors to help explain short and intermediate fluctuations, he still viewed entrepreneurial-induced technological and organizational change as the 'dominant element' whose economic impact would determine the contours of shorter cycles.

Schumpeter (1939, pp. 161–162) also characterized business fluctuations as one where 'many simultaneous cycles can co-exist such that there is 'no reason why the cyclical process of evolution should give rise to one wavelike movement'; charts of actual economic and financial time series presented in the text support the assumption of 'the presence of many fluctuations, of different span and intensity, which seem to be superimposed on each other.' Thus, it is precisely this idea of 'superimposition' (Geiger 2014) that links together the empirical and graphical approaches used by Kondratieff and Schumpeter with modern UCMs to identify and estimate the periodicity of long cycles. Nevertheless, the main advantage of a UCM over a time-trend regression model or other low-frequency detrending techniques (e.g., the Hodrick-Prescott lowpass filter) is the ability to *simultaneously* estimate the short, intermediate, and long cycle periodicity by incorporating fixed and stochastic trend and cycle parameters. A second advantage is that UCMs do not require any arbitrary transformation of a time series (e.g., first-differencing) to ensure stationarity of a time series that is required for both ARIMA modeling and spectral analysis.[13] Finally, a UCM produces 'damping factors' to help determine the endogeneity of a particular cycle. Thus, the UCM presents a more general and flexible model for extracting long wave periodicities from economic and financial time series.[14]

The next section describes the 'unobservable components' in Kondratieff and Schumpeter's long wave methodology; Section Three replicates and evaluates Kondratieff's econometric time-trend models; Section Four calculates and compares implied periodicities based on Kondratieff's published years of peaks and troughs as well as those generated by his econometric time-trend models as well as UCMs; Section Five evaluates UCM model diagnostics; Section Six compares long cycle periodicities, the UCM measure of endogeneity and intermediate and short cycle periodicities; Section Seven presents

[13]Spectral analysis has been the most common empirical method for identifying long waves because it can simultaneously decompose any time series into cyclical components with different frequencies (Kuczynski 1978; van Ewijk 1982; Metz 2009).

[14]Please see Appendix 2 for technical details on the UCM estimation procedures.

new preindustrial long cycle estimates based on recent long-term data compiled by Thomas and Dimsdale (2017) at the Bank of England (BOE) entitled 'A Millennium of UK Data'; and Section Eight provides a summary of the main conclusions. Two technical appendices describe the classical trend-cycle decomposition as well as the UCM methodology.

2. Kondratieff and Schumpeter's 'Unobservable Components'

Economists have long relied upon quantitative methods to extract and identify latent or 'unobservable' components from economic and financial data. 'Fitting' historical data to a time-trend regression model and retrieving the predicted values is a standard statistical technique to obtain the trend values embedded in times series observations. Although Klein (1997, p. 230) cites Hooker (1901) as the analyst who 'coined the term "trend,"' their methodology relied on correlation coefficients and not the estimated slope from an econometric time-trend model. Kondratieff (1925, p. 579) recognized the importance of equating the predicted or 'unobservable' trend values from a least-squares regression model as an expression of the 'evolutionary (or non-reversible) processes' of a capitalist economy. For example, Kondratieff (1925, p. 580) viewed the trend rate of population growth as a non-reversible process compared to cyclical or 'reversible values':

> By 'wavelike' or 'fluctuating' processes are meant processes of variation which are changing their direction in the course of time and subject to repetition and reversion. Such are changes in prices, in the rate of interest, in the percentage of unemployed. … Considered as continuous, the processes of change may be represented by curves whose directions and slopes vary, exhibiting a series of recurring maxima and minimum.

Kondratieff's identification of a long wave in a specific economic series was based on the wavelike pattern formed by the nine-year moving average residuals ('deviations') from a time-trend regression model. Kondratieff (1979, p. 522), characterized the smoothed values as a 'theoretical data series … which accurately enough expresses the general direction of the empirical [i.e., actual] series.' In another translation of the *RES* article, Kondratieff (1984, p. 36) stated that the fitted vales were employed to 'eliminate the influence of intermediate cycles whose average length is about nine years … .' However, the *published* year of a specific long wave peak and trough were based on the *actual* maximum and minimum values, whereas the charts of long waves displayed in the article were based on the smoothed residuals from time-trend regression models. This discrepancy was underscored in a footnote to the first table of the 1979 and 1984 versions of the original paper and suggests some ambiguity about the precise method used to date long wave turning points.[15] Figure 1 displays the wholesale price index (WPI) or 'index of commercial prices, expressed in terms of gold (1901–1910 = 100)' for England, France

[15]The confusion appears to arise from earlier English translations of the original Russian text by Kondratieff (1926). Footnote 21 in Kondratieff (1979, p. 532) states that: 'Table 1 enumerates the maxima and minima according to the original data. The problem of the most *accurate* method for the determination of the maxima and minima would deserve a special analysis; at present *we leave the question open*' (emphasis added. In Kondratieff (1984, p. 61) the note below Table 1 reads as: 'The maxima and minima given in the table are based on unsmoothed series. The question, however, *as to the method* of determining the maxima and minima should be accorded special analysis. But for the time being *I leave it open*. In view of that fact, I regard the turning points indicated in the table as only the most probable and the *closest to the real one*' (emphasis added). In the latest and most complete published version of the article (Kondratieff 1926 [1998]), there is no footnote below Table 1.

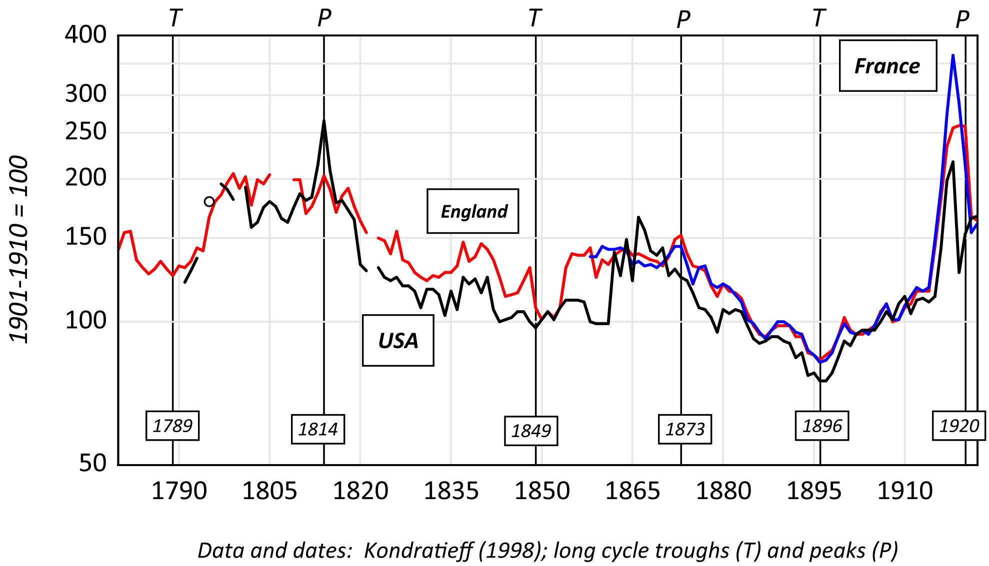

Figure 1. Wholesale price index per ounce of gold.

and the USA.[16] In Kondratieff (1926 [1998], p. 37) the long wave peaks (P) and troughs (T) are also shown for England based on dates identified in Table 1 of the text.

It is interesting to note that this graph also appears in the 1979 version of the original *Review of Economics and Statistics* article, however, straight lines drawn between the dates of peaks and troughs were superimposed on the graph. How exactly Kondratieff generated these lines is never stated, however, they strongly suggest that the fitted values were generated by separate time-trend models estimated between the trough and peak years. This graph also underscores the advantages of UCMs to integrate different trend and cycle estimates.[17]

Schumpeter understood that a long wave could not be identified in an individual time series unless there was a valid and reliable method for separating the secular trend from the cyclical component(s). Having written the introductory essay for the first issue of *Econometrica*, Schumpeter was likely to be familiar with technical issues and problems associated with trend-cycle decompositions. For example, in his review of W.C. Mitchell's *Business Cycles* (1927), Schumpeter (1930, p. 166) presciently observed how 'trend analysis will be the central problem of our science in the immediate future and the center of our difficulties as well.'[18] Furthermore, some analysts were skeptical about the use of moving averages to analyze cyclical movements in time series data. For example, several leading statisticians in the 1920s and 1930s demonstrated how

[16]All observations are from data tables in Kondratieff (1926 [1998]).

[17]Harvey's (1989) 'Structural Time Series' (STS) model forms the basis for the UCMs used in this paper. Among Harvey's innovations is the flexibility to incorporate fixed and stochastic parameters into trend and cycle estimations. As Goldstein (1999, p. 72) observed: 'The pliability of each component derives from the incorporation of stochastic elements, but in a manner that preserves a *deterministic* representation as a *special case*' (emphasis added). See appendix 2 for further details.

[18]Makasheva (2022, p. 277), observed that 'He [Kondratiev] stood closer to Kuznets' position — at least to his view on the trend as a theoretical problem. … We are not aware to the extent that Kondratiev relied upon Kuznets' work, but in any case, we may say that Kondratiev and Kuznets shared the interest in the problem.'

moving averages could generate spurious correlations. Foremost among them was the Russian economic statistician and Kondratieff's subordinate at the *Moscow Conjuncture Institute*,[19] Eugen Slutsky[20] who proved that a moving average of a random series could generate a cyclical pattern with a regular sinusoidal periodicity (Klein 1997, p. 277).[21] Nevertheless, Slutsky (1927 [1937], quoted in Klein 1997, p. 278) recognized that economic and financial time series also reflected fundamental technical, social, and other forces dynamic phenomena worthy of study:

> Those investigators of economic life are right who believe in their acumen and instinct and subscribe to at least an approximate correctness in the concept of the periodicity of business cycles.

Notwithstanding Schumpeter's elegant trigonometric presentation of business cycles, he never applied statistical techniques to empirically validate his theoretical specifications.[22] Instead, he presented many graphs and charts throughout *Business Cycles* to illustrate periodicities of differing lengths, including the beginning of a third Kondratieff long wave (see pp. 397–436). For example, on page 686 Schumpeter references Chart XXXVI (a graph of several financial time series from 1866 through 1914) that implicitly 'fits' — without showing any statistical measures of association — the short (Kitchin), intermediate (Juglar), and long wave (Kondratieff) movement in US stock prices:[23]

> In both railroad and industrial stock prices the Kondratieff prosperity from 1897 on shows well and so do the Kitchins. The major movements which we observe, however, clearly reflect the Juglars: we see the (anticipating) boom of 1868 and 1869 and then the characteristic slump from 1873 to 1877; then (also anticipating) boom from 1877 to 1881; the same phenomena, most regularly repeated from 1885 on; no such precedence for the first Juglar of the third Kondratieff, which may have been due to the aftereffects of 1893 and political factors; but more regular behavior again in the second. This reflects the relation, which according to our analysis should exist between speculation and the investment process. The latter, as we know, *dominates the Juglars more obviously than the Kitchins*. (Emphasis added)

Thus, Schumpeter believed that slower-moving fundamental factors such as capital investment would regulate movements in more volatile financial measures such as stock prices. Given his theoretical model, Schumpeter also expected a hierarchy of

[19]Makasheva (1998, p. xxix) observed that in October 1920, Kondratieff was appointed the first Director of the Conjuncture Institute whose research on business and agricultural cycles was 'known and highly evaluated by Keynes, Mitchell, Fisher, and other prominent economists of the period. Kondratieff's personal contribution to economics was fully acknowledged and he was elected to several well-known foreign scientific societies including the American Economic Association, the American Association of Agricultural Economics, the Royal Economic Society, and the Royal Statistical Association; he was also included on the editorial boards of several leading journals.'

[20]Slutsky (1915 [1952]) is best known for his formal demonstration of the income and substitution effect arising from a change in price. However, as Lenfant (2022, p. 208) notes, Slutsky's original (1927 [1937]) contributions to the theory of random fluctuations demonstrated how a 'random time series, under the effects of some lagged and weighted composition rule can generate correlated series of values that exhibit non-erratic wave movements.' Klein (1999, p. 160) shows how Slutsky made significant contributions to the mathematics of stochastic processes and meteorology which helped him avoid Kondratieff's fatal demise in Stalin's gulags.

[21]Similarly, Yule (1926) demonstrated how 'nonsense' or spurious correlation could persist in non-stationary (i.e., trended) but unrelated time series. Granger and Newbold (1974) expanded on the phenomena of spurious correlation and regression using Monte Carlo simulations.

[22]Of the sixty graphs displayed in *Business Cycles*, only the first two are based on his trigonometric equations.

[23]None of the graphs in this chapter included a label for the *y*-axis. In other chapters, the *y*-axis is labeled as a percent change; for data expressed in levels, Schumpeter uses a semi-log scale to capture relative growth and remove distortions caused by monotonic movements in a time series.

cycles with differential durations where shorter-term phenomena such as stock prices are embedded within intermediate-term capital investment cycles which are themselves embedded in long wave movements of technological, organizational and other epiphenomenal changes. Unfortunately, Schumpeter was never able to provide any statistical or econometric support for his descriptions and depictions of overlapping cycles with differing periodicities.

3. Replicating Kondratieff's Econometric Time-Trend Models

Since Kondratieff never described precisely how he calculated the periodicity for a given time series, we must follow his implied method of using *actual* values of a time series to identify long cycle peak and trough. We then calculate the average length of a long cycle from successive peak-to-peak years and trough-to-trough years, respectively. Trying to reconcile these implied periodicities with those calculated from econometric predictions of long cycle peak and trough years is not without risk: on the one hand, Kondratieff (1926 [1998]) identified the specific year of a long cycle peak and trough based on the highest and lowest *actual* values in a given time series. On the other hand (p. 31), he was *unequivocal* in stating that the best statistical evidence for *identifying* a long cycle in any time series was based on the smoothed residuals from an econometric time-trend model:

> To identify long cycles *explicitly*, I took another step in the data processing and smoothed the series of deviations obtained by a moving average method. Here, to smooth out and eliminate the effect of medium-length cycles, with duration, equal on average to approximately 9 years. However, in this way, I at the same time eliminated not only the effect of medium-length cycles but also that of small cycles (if there were any) and that of random fluctuations. (Emphasis added)

Kondratieff's statistical method for extracting the trend from the long cycle for any given time series, was based on a two-step procedure. The first step was to generate a 'theoretical curve' based on the predicted values from a single or higher-order (polynomial) econometric time-trend model.[24] Kondratieff (p. 30) acknowledged that his trend estimates might or might not 'correspond to real general evolutionary trends in the development of the economy,' however, 'the nature of the theoretical curve' must be the 'subject of further work' The second step involved calculating centered nine-year moving averages of the residuals whose maximum and minimum values would indicate the peak and trough years of a long cycle. Kondratieff's published figures included a plot of the actual and predicted (trend) values as well as separate plots of the residuals ('empirical deviations') and smoothed residuals ('smoothed deviations').[25]

[24]Kondratieff (1926 [1998], p. 30) justified this method because it was 'in accordance with the methods of mathematical statistics, [such that] I construct a theoretical curve which reflects the general direction of the main trend in the empirical series sufficiently accurately … I avoided curves of very high degree. Nevertheless, it was sometimes decided to use a parabola of degree three. Of course, over such a long period, for example of a 100 years or more, it is not easy to find this theoretical curve. Thus, great attention was paid to this in the work.'

[25]Replicating Kondratieff's econometric models is important because he sometimes selected peak and trough years that were outside the historical/estimation periods. Furthermore, a complete record of the actual, predicted and smoothed residual values has only been available since the 1998 complete version of the *RES* paper. Finally, Kondratieff never explained how/if he incorporated or applied any model results to estimating periodicities.

Table 1 compares Kondratieff's (1926 [1998]) time-trend coefficients with updates produced by the EViews econometric software program. Most of the original and revised coefficients were close in value with differences likely due to computational error and/or algorithmic precision; exact matches of coefficients are bolded while associated summary measures of statistical significance are denoted with asterisks. The last two columns report the adjusted R-squares and DW statistics for the EViews equations. Though the Durbin-Watson statistic did not exist in Kondratieff's day, it is not surprising that the time-trend models suffer from first-order serial correlation.[26]

Kondratieff (1926 [1998]) estimated three additional econometric time-trend models that were omitted from the *RES* article. Row (14) shows the model results for aggregate end-of-year balances of French private bank deposits from 1835 through 1913. The model of annual wages of workers in the French coal mining sector from 1847 through 1913 is shown in row (15). Kondratieff (1935) published the peak and trough years for this variable, however, no information about the econometric model was revealed until the 1998 article. Row (16) displays the model coefficients for French consumption of mineral fuels from 1827 through 1913 that also produced an exact match of almost every polynomial time-trend coefficient. However, Kondratieff never published peak and trough years for this variable.

The econometric replication exercises reveal that Kondratieff and his staff were able to carry out complex, labor-intensive mathematical calculations to many significant digits in order to generate regression results that are quite close (and sometimes exactly equal to) those produced with the EViews econometric software program. Secondly, despite the absence of significance tests and goodness-of-fit measures, most of Kondratieff's time-trend model specifications included statistically significant slope coefficients.

4. Comparing Kondratieff's Long Wave Turning Points and Implied Periodicities

Table 2 compares estimates of Kondratieff's long cycle periodicities based on: (1) published dates of turning point years; (2) time-trend regression models; and (3) UCMs. For each time series, the row labeled 'Published' shows the peak and trough years listed in Table 1 of the 1998 version of Kondratieff's original paper.[27] The next row, 'Time-trend model' makes the same calculation based on the peak and trough years of the smoothed residual values generated by the updated EViews models.[28] The row labeled 'UCM' generates a third estimate of a long wave periodicity based on the predicted long cycle peak and trough years.[29] The last three columns calculate the implied periodicities based on the average difference between successive peak-to-peak years, trough-to-trough years (produced by each modeling technique), and an average of both of these values.

Rows (1) through (3) in Table 2 display the WPI long wave turning point years for England, France and the USA. The average WPI long cycle periodicity for England

[26]Kondratieff did not publish any goodness-of-fit, t-statistics or other diagnostic information.
[27]Turning point years rarely deviated across the 1935, 1979, 1984 and 1998 published versions of the paper.
[28]Calculating nine-year centered moving averages necessitates the loss of the first four and last four smoothed residuals.
[29]The STAMP software program produces predicted values for each cycle without the loss of the first and last four observations.

Table 1. Comparison of Kondratieff's econometric models.

Row		*Estimation Perod*	Kondratieff (1926 [1998])				EViews 10			
			β_0	t	t^2	t^3	β_0	t	t^2	t^3
	WPI † (Table 1)									
(1)	England	1780–1914	139.00	−1.113	−0.0028	0.00020	138.94	−1.113***	−0.00258**	0.00019***
(2)	France	1858–1914	No model was specified.				103.78	−1.910***	0.03574***	0.00215***
(3)	USA	1791–1914	No model was specified.				115.31	−0.695	0.00694***	-
	Bond Indices									
(4)	England (Consols — Table 2)	1816–1922	112.57	0.260	−0.01200	−0.00020	94.81	−0.214	0.02060***	−0.00020***
(5)	France (Annuities — Table 3)	1814–1922	78.99	0.230	-	-	66.31	0.235***	-	-
	English Nominal Wages									
(6)	Cotton-Textile Industry (Table 4)	1807–1913	64.128	1.053	0.0099	−0.00023	121.094	−3.810	0.05860***	−0.00023***
(7)	Agricultural Laborer (Table 5)	1789–1913	91.587	0.454	-	-	88.400	0.316***	-	-
	Foreign Trade									
(8)	External Trade Turnover — England (Table 6)	1802–1914	1.0293	0.0096	−0.000060	-	1.0756	0.0090***	−0.000060***	-
(9)	External Trade Turnover — France (Table 7)	1827–1913	146.39	3.46	0.006	-	146.39	3.46***	0.006***	-
	Per Capita Coal Production and Demand									
(10)	Production Per Capita — England (Table 8)$^{\tau}$	1855–1917	3.6614	0.0063	−0.000094	-	2.9672	0.0172***	−0.000093***	-
(11)	Consumption Per Capita — France (Table 9)	1827–1913	539.21	16.900	0.13260	0.00003	538.96	16.905***	0.13283***	0.00026
	English Per Capita Production									
(12)	Cast Iron (Table 10)	1839–1914	193.30	2.28	−0.556	-	193.76	2.238***	−0.0544***	-
(13)	Lead (Table 11)$^{\tau}$	1855–1920	0.0278	−0.0166	−0.0012	-	0.0361	−0.0165***	−0.00012***	-
(14)	French Private Savings Deposits (Table 12)	1827–1913	1133.90	57.227	0.7724	-	1123.39	57.787***	0.8050***	-
(15)	Annual Wages of French Coal Workers (Table 4)§	1827–1913	1012.28	14.23			1013.78	14.36***	-	-
(16)	Consumption of Mineral Fuel in France‡	1827–1913	539.21	16.900	0.133	0.00026	539.25	16.900***	0.132***	0.00026***

†Index of commercial prices, expressed in terns of gold.
$^{\tau}$Log $_{10}$ Transformation.
§Supplemental table in 'D.I Oparin's additional material'
‡Omitted from Kondratieff (1935) and only in (1984).
**Significant at the 0.05 confidence level.
***Significant at the 0.01 confidence level.
Source: Appendix tables, Kondratieff (1926 [1998]) and Author's calculations.

Table 2. Comparative dates of Kondratieff long cycle peaks (P) and troughs (T).

Row		Estimation Period	Model	First Cycle		Second Cycle		Third Cycle		Average Periodicity		
				T	*P*	*T*	*P*	*T*	*P*	Peak-to Peak	Trough-to Trough	Overall
(1)	England	1780–1914										
			Published	1789	1814	1849	1873	1896	1920	53.0	53.5	53.3
			Time-trend model*	1788	1803	1830	1873	1895	1920	58.5	53.5	56.0
			UCM**	1780	1809	1839	1868	1898	1925	58.0	59.0	58.5
(2)	France	1858–1914										
			Published	n/a	n/a	n/a	1873	1896	1920	47.0	n/a	47.0
			Time-trend model	n/a	n/a	n/a	1875	1886	1916	41.0	n/a	41.0
			UCM**	n/a	n/a	n/a	1860	1898	1918	58.0	n/a	58.0
(3)	USA	1791–1914										
			Published	1790	1814	1849	1866	1896	1920	53.0	53.0	53.0
			Time-trend model	1795	1814	1832	1866	1895	1921	53.5	50.0	51.8
			UCM**	1791	1808	1838	1867	1897	1925	58.5	53.0	55.8
	Bond Indices											
(4)	England (Consols)	1816–1922										
			Published	1790	1816	1844	1874	1897	1920	52.0	53.5	52.8
			Time-trend model	n/a	1820	1842	1868	1898	1917	48.5	56.0	52.3
			UCM	n/a	1816	1837	1866	1895	1922	53.0	58.0	55.5
(5)	France (Annuities)	1814–1922										
			Published	n/a	1816	1844	1872	1894	1921	52.5	50.0	51.3
			Time-trend model	n/a	1818	1842	1871	1896	1918	50.0	54.0	52.0
			UCM	n/a	1814	1839	1868	1898	1922	54.0	59.0	56.5
	English Nominal Wages											
(6)	Cotton-Textile Industry	1807–1913										
			Published	n/a	1810	1850	1874	1890	1921	55.5	40.0	47.8
			Time-trend model	n/a	1818	1851	1873	1887	1909	45.5	36.0	40.8
			UCM	1807	1827	1851	1872	1892	1913	43.0	41.0	42.0
(7)	Agricultural Laborers	1789–1913										
			Published	1790	1817	1844	1875	1889	1921	52.0	49.5	50.8
			Time-trend model	1793	1807	1848	1875	1886	1909	51.0	46.5	48.8
			UCM	1789	1810	1845	1874	1893	1913	51.5	52.0	51.8
	Foreign Trade											
(8)	External Trade Turnover — England	1802–1914										
			Published	n/a	1810	1842	1873	1894	1920	55.0	52.0	53.5
			Time-trend model*	n/a	1806	1838	1872	1896	1920	57.0	58.0	57.5
			UCM	n/a	1802	1837	1874	1896	1924	61.0	59.0	60.0

(*Continued*)

Table 2. Continued.

Row		Estimation Period	Model	First Cycle T	First Cycle P	Second Cycle T	Second Cycle P	Third Cycle T	Third Cycle P	Average Periodicity: Peak-to Peak	Average Periodicity: Trough-to Trough	Average Periodicity: Overall
(9)	External Trade Turnover — France	1827–1913										
			Published	n/a	n/a	1848	1872	1896	1920	48.0	48.0	48.0
			Time-trend model***	n/a	1835	1854	1883	1909	1913	39.0	55.0	47.0
			UCM	n/a	1827	1850	1876	1901	1924	48.5	51.0	49.8
	Per Capita Coal Production and Demand											
(10)	Coal Production — England	1855–1917										
			Published	n/a	n/a	1850	1873	1893	1914	41.0	43.0	42.0
			Time-trend model	n/a	n/a	1859	1869	1894	1910	41.0	35.0	38.0
			UCM	n/a	n/a	1855	1867	1890	1912	45.0	35.0	40.0
(11)	Coal Consumption — France	1827–1913										
			Published	n/a	n/a	1849	1873	1896	1914	41.0	47.0	44.0
			Time-trend model	n/a	1835	1852	1874	1893	1913	39.0	41.0	40.0
			UCM	n/a	1827	1848	1874	1896	1913	43.0	48.0	45.5
	English Per Capita Production											
(12)	Cast Iron	1840–1914	Published	n/a	n/a	n/a	1871	1891	1914	43.0	n/a	43.0
			Time-trend model*	n/a	n/a	1843	1874	1894	1909	35.0	51.0	43.0
			UCM****	n/a	n/a	1850	1871	1893	1913	42.0	43.0	42.5
(13)	Lead	1855–1920	Published	n/a	n/a	n/a	1870	1892	1914	44.0	n/a	44.0
			Time-trend model	n/a	n/a	1859	1869	1901	1915	46.0	42.0	44.0
			UCM	n/a	n/a	1855	1874	1894	1914	40.0	39.0	39.5
(14)	French Private Savings Deposits	1827–1913	Published	n/a	n/a	1844	1874	1892	n/a	n/a	48.0	48.0
			Time-trend model	n/a	n/a	1843	1874	1894	1913	39.0	51.0	45.0
			UCM	n/a	n/a	1846	1874	1895	1913	39.0	49.0	44.0
(15)	Wages of French Coal Workers	1827–1913	Published	n/a	n/a	1849	1874	1895	n/a	n/a	46.0	46.0
			Time-trend model	n/a	n/a	1852	1876	1895	1913	37.0	43.0	40.0
			UCM	n/a	n/a	1857	1877	1898	1913	36.0	41.0	38.5
(16)	Per Capita Consumption of Mineral Fuel in France‡	1827–1913	Published	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
			Time-trend model	n/a	n/a	1849	1870	1890	1913	43.0	41.0	42.0
			UCM	n/a	n/a	1848	1872	1895	1913	41.0	47.0	44.0

*Time-trend model residuals were extrapolated from 1915 to 1924.
**UCM estimation period through 1925.
***Time-trend model residuals were extrapolated from 1914 to 1924.
****UCM estimation period through 1924.
‡No published turning point years; data and equations from Kondratieff (1984, pp. 126–127).
Source: Kondratieff (1926 [1998], p. 37) and Appendix tables; Author's calculations.

and the USA are bounded between about 53 and 58 years while the French WPI periodicities range between 41 to 58 years. Kondratieff's published turning points for the first and second long wave cycles show some interesting divergences during the first half of the 19th century. For England, Kondratieff's published first-cycle peak occurred in 1814, however, the time-trend smoothed residuals peak in 1803 or eleven years before the published peak. The second-cycle trough had a similar discrepancy with the published trough (1849) occurring 19 years after the time-trend trough (1830). In both cases, Kondratieff (1926 [1998], p. 31) seems to have ignored the smoothed residuals and simply determined the WPI turning points from Figure 1 which 'shows quite clearly that despite all the deviations and irregular movements, the average level of commodity prices exhibits a series of long cycles.'[30] The first published trough year for the USA WPI was in 1790, however, Kondratieff must have selected this year because his actual time series commenced in 1791. Thus, it is not surprising that the trough year predicted by the time-trend model (1795) and UCM (1791) were slightly later. The published peak year (1814) for the first long wave was much closer to that produced by the time-trend model (1814) and UCM (1808). Kondratieff' identified 1849 as the trough year for the WPI in both England and the USA. Once again, for England, earlier second-cycle troughs were predicted by both the time-trend model (1830) and UCM (1839); for the USA the corresponding WPI modeled trough years were in 1832 and 1838, respectively. Thus, the greatest discrepancy occurred between the published and modeled trough year for the first complete long wave cycle through 1850.[31] In sum, Kondratieff's published turning points for the WPI do not appear to have been influenced by predictions produced by his econometric time-trend models.

Figures 2–4 display the four-graph format produced by the STAMP UCM program for the WPI for England, France and the USA, respectively.[32] The top-left graph of Figure 2 plots the actual and trend values of the WPI, followed — in a clockwise direction — by plots of the extracted short, long, and intermediate cycle components. The UCM long cycle graph for England ('wpieng-Cycle3') displays two and a half complete long cycles of a little more than 50 years per cycle.

Figure 2 displays the UCM results for France where the graphical evidence is less compelling, partly because the WPI data start in 1858. The top left-side graph shows the actual data with the superimposed trend component.[33] For example, the French WPI trend is flat and specified with a trend order equal to 1, while that for England and the USA is curvilinear with trend orders equal to 3 and 2, respectively.[34]

A comparison of English (Consols) and French (Annuities) bond index turning points are shown in rows (4) and (5) of Table 2. The long cycle average periodicities are bounded between 51 and 57 years, though Kondratieff did not publish a first-cycle trough date for France. Kondratieff (1926 [1998], pp. 32–33) attributed the first long

[30]Since Kondratieff did not estimate econometric models for the WPI for France and the USA, he may have decided to ignore the turning points indicated by the smoothed residuals.
[31]The second cycle published WPI peak year for England (1873) and the USA (1866) were much closer to the corresponding modeled peak years.
[32]The next section of the paper evaluates the individual UCM diagnostics for the models that generated each graph.
[33]STAMP permits the trend component to be specified as a non-linear polynomial function with an 'order' ranging from 1 to 4. See Table 3, column 7 for the complete model specification.
[34]Similar UCM graphs for all variables are available from the author.

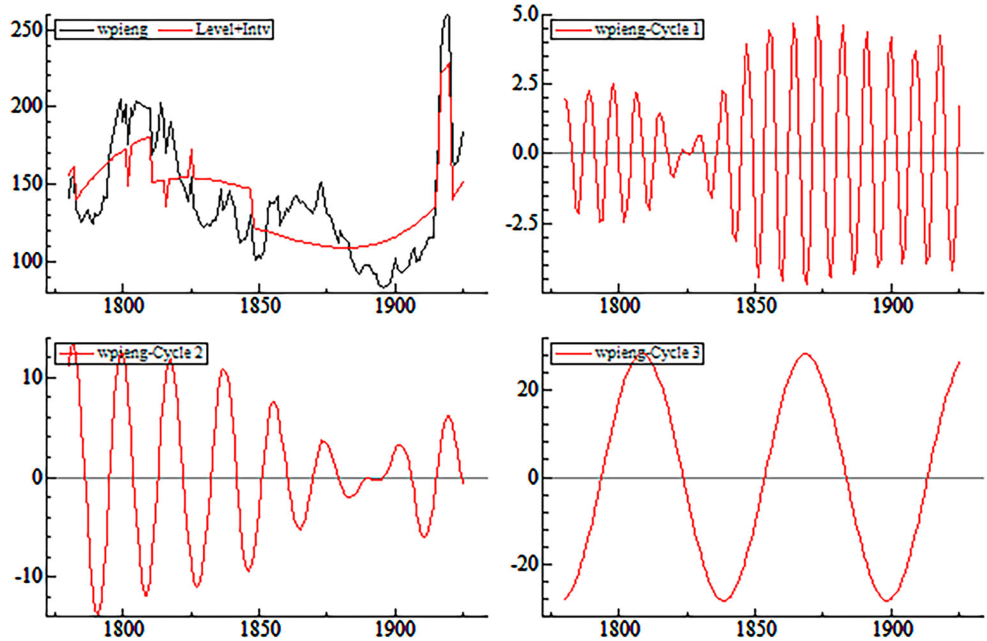

Figure 2. Index of commercial prices for England: 1780–1914. Source: Kondratieff (1998) and author's calculations.

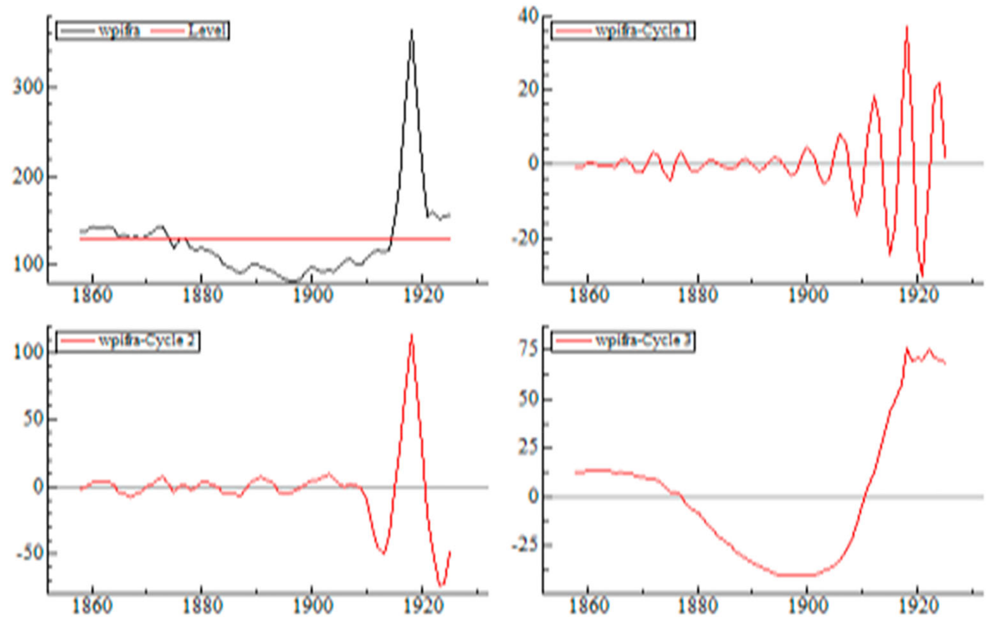

Figure 3. Index of commercial prices for France: 1858–1914. Source: Kondratieff (1998) and author's calculations.

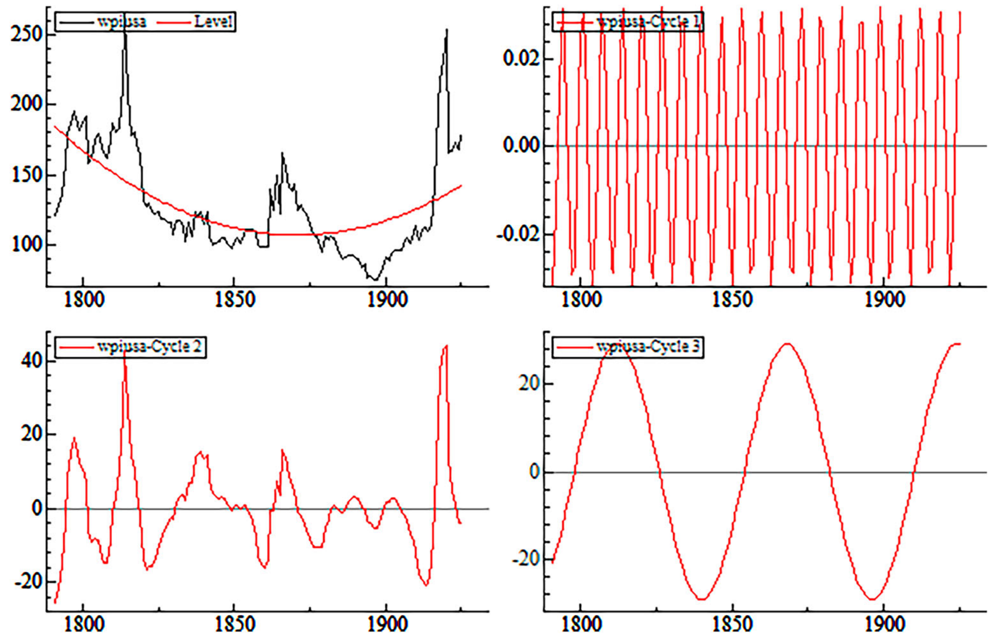

Figure 4. Index of commercial prices for the USA: 1791–1914. Source: Kondratieff (1998) and author's calculations.

wave cycle in interest rates to the rise and fall of commercial prices during the Napoleonic wars, and remarks upon the (inverse) association over the entire estimation period:

> It is clear from these figures that long cycles are present in the movement of the rates of bonds, and, consequently, in interest rates on capital. … The presence of long cycles in the movement of interest rates on capital is thus revealed in all its clarity. The period of these cycles very nearly coincides with the corresponding periods in the movement of commercial prices.

Reproducing Kondratieff's tuning point years for nominal wages of English cotton textile workers posed a challenge for both the time-trend and UCM specifications. Row (6) in Table 2 shows that Kondratieff only published an estimate of the first cycle peak (1810) while the third-cycle peak year (1921) lies eight years beyond the time-trend model estimation horizon; the range of long cycle average periodicities was bounded between 41 and 48 years. Row (7) shows the peak and trough years for the nominal wages of English agricultural workers which have implied long cycle average periodicities between 49 and 52 years. Overall, Kondratieff (1926 [1998], pp. 32–33) was quite sanguine about the evidence of *coincident* long cycles in his wage, price, and interest rate data:

> Thus, despite the scarcity of information about wages, long cycles are undoubtedly observed in their movements. The periods of these cycles almost coincide with the periods of the cycles for commercial prices and interest rates on capital.[35]

[35]The correlation of smoothed residuals between the English WPI and English bond index is (−0.93); for the equivalent French data it is only (−0.14).

Kondratieff's estimate of the average full cycle periodicity for England's external trade turnover of 53.5 years is 4.0 and 6.5 years, respectively shorter than the time-trend and UCMs estimates, respectively (see the last column in row (8)). In addition, the last published peak year (1920) lies outside the estimation period of Kondratieff's time-trend model.[36] By using the actual observations to date the first peak year, Kondratieff appears to have ignored the smoothed time-trend model residuals which generate a more elongated first long cycle and longer average periodicity. Row (9) compares turning point years and periodicities for France's external trade turnover. The average periodicity based on the published, time-trend and UCM are 48.0, 47.0, and 49.8 years, respectively; however, the estimation period (1827–1913) only covers two complete long cycles. While Kondratieff (1926 [1998], p. 34) acknowledged the absence of data to support a first long wave, he noted: 'Thus, the data about foreign trade convincingly reveal two long cycles, where the periods of these cycles … are very similar to the periods of cycles apparent in other data.'

Per capita coal production and consumption in England and France, respectively are shown in rows (10) and (11) of Table 2. For England, the average periodicities ranged between 38.0 and 42.0 years across all three approaches. For the time-trend model, Kondratieff converted the dependent variable (i.e., the level of English coal production) to base-10 values and then estimated a quadratic time-trend model to capture the non-linear pattern of English coal production. For French coal consumption, the average long cycle periodicities were clustered between 40.0 and 45.5 years. Kondratieff (1926 [1998], p. 35) underscored the need to verify long cycles in 'purely real elements' (e.g., coal, cast iron, and lead) and commented that despite their shorter time series 'in the data about the dynamics of the rate of extraction and consumption of coal we clearly observe almost two long cycles.' The finding of long cycles in coal production and consumption contradicts D.I. Oparin's claim that Kondratieff's measures of output and production do not contain long cycles.[37]

Rows (12) and (13) in Table 2 display the turning point years for per capita English cast ('pig') iron and lead production. Data on the former variable begin in 1840 and Kondratieff (1926 [1998], p. 35) briefly comments that 'one and a half cycles can clearly be seen … ' Kondratieff's published and time-trend models produced the same implied average periodicities for English cast iron (43.0 years) and lead production (44.0 years); the UCM estimates were 42.5 and 39.5 years, respectively. Once again, Oparin's claim that long cycles are not present in Kondratieff's output measures is refuted.

Kondratieff provided no commentary about the long cycle periodicities for three French variables with published time-trend coefficients: savings deposits, coal miner wages and mineral fuel consumption. Since there is no data for any of these variables before1827, it was not possible to identify peak and trough years during the first long cycle. Row (14) displays the average long wave periodicities for deposits at French public savings banks. Using Kondratieff's published turning point dates, the average periodicity of 48.0 years was close to the time-trend and UCM average periodicities of 45.0

[36]Although Kondratieff (1926 [1998]) displayed historical data through 1924, he used the model coefficients estimated from 1802 through 1914 to generate *ex post* (in-sample) predicted and smoothed residuals for the period from 1915 through 1924. No explanation was given for the use of a truncated estimation period.

[37]See Garvey (1943, p. 211). Oparin (1926 [1998]) was a colleague of Kondratieff's at the *Moscow Conjuncture Institute* who also presented a counter-response to Kondratieff (1926) at the *Institute for Economics*.

and 44.0 years, respectively. Row (15) shows that the implied average long cycle periodicities for French coal miner wages across all three estimates were bounded between 38.5 and 46.0 years. Kondratieff did not publish turning points for French consumption of mineral fuels in any version of his long cycle paper. Row (16) shows the implied average periodicities generated by the re-estimated time-trend and UCM of 42.0 and 44.0 years, respectively. To our knowledge, this is the first attempt to replicate and analyze these particular variables and models.

The length of the average long cycle based upon Kondratieff's published turning points across all variables is 47.9 years which is close to the average periodicity produced by the time-trend (45.2 years) and UCM models (46.6 and 49.3 years).[38] These results provide support for Schumpeter's expectation of a fifty-year long cycle as well as his theoretical periodicities for an intermediate and short cycle.[39]

5. Evaluation of Kondratieff's UCMs

Table 3 provides a summary of UCMs (based on Kondratieff's original data) including diagnostic statistics, trend specification and convergence strength. Column (1) shows the conventional adjusted R-square, while column (2) *R-diff* is the random-walk R-square which shows the percentage improvement in fit over a random walk plus drift model.[40] Column (3) is the standard Durbin-Watson statistic; column (4) *pev/md* is the ratio of the 'prediction error variance' to the mean deviation of the residuals where 'in a correctly specified model the reported ratio should be close to unity' (STAMP 2009, p. 206); column (5) indicates whether an AR(1) term is included in the model. Additional UCM trend specification and convergence criteria are displayed in the following four columns: columns (6) and (7) indicate whether the variance of the level (y-intercept) and slope is specified as a fixed or stochastic parameter; column (8) displays the STAMP nomenclature to denote the trend based on the combination of level and slope parameters; column (9) indicates the strength of the maximum likelihood model convergence.[41]

The diagnostic statistics for the UCMs are generally very good. The adjusted R-squares are mostly above ninety percent while the generally low *R-diff* statistics support the UCM specification over a random walk model. The DW statistics indicate no significant serial correlation which suggests that the models are able to adequately capture the dynamic structure of each time series. Combining this inference with the fact that the *pev/md* ratios are generally close to unity underscores the adequacy of the trend and other component specifications for almost every UCM.[42] Columns (6), (7), and (8) indicate that the longer time series were best modeled with 'deterministic' trends while the shorter time series required a stochastic trend specification for the level and/or slope.[43] Finally,

[38]See Table 4, bottom row 'Average.'
[39]See footnote 3.
[40]*R-diff* is a measure of significant trend movement which is defined as the ratio of the residual variance from a random walk with drift specification to the variance of first differences (Koopman et al. 2009, pp. 154 and 206).
[41]As Koopman et al. (2009, p. 153) note: 'The most important piece of information is the message "Very strong/Strong/Weak/No convergence in … iterations. The precise definition of these terms may be found in § 9.6, but from a practical point of view, the appearance of the word "strong" is to be welcomed.'
[42]The highest *pev/md* values are for France's WPI (2.92) and External Trade Turnover (3.56). The WPI value may be due to a relatively short time series that starts in 1858; however, an explanation for the latter value is not obvious.
[43]Goldstein (1999) modeled long waves in real per capita GDP and found that in 9 of 11 advanced capitalist economies, the 'dominant' level and slope trend UCM specification was either fixed/fixed or stochastic/fixed.

Table 3. Kondrtaieff UCM diagnostic statistics and trend specifications.

Row	Columns	Period	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					Diagnostic Statistics				UCM Trend Specification		Model
	WPI [†]		R^2	*R-diff*	DW	pev/md	AR(1)	Level	Slope	Description	Convergence
(1)	England	1780–1925	0.984	0.870	1.66	1.04	Yes	F	F(3)	Deterministic; No Drift	Strong
(2)	France	1858–1925	0.862	0.389	1.78	2.92	Yes	F	No	Deterministic; No Slope	Strong
(3)	USA	1791–1925	0.865	0.186	1.78	1.77	Yes	F	F(2)	Deterministic; No Drift	Strong
	Bond Indices										
(4)	England (Consols)	1816–1922	0.894	0.126	1.53	1.15	Yes	S	F	Local Level With Drift	Very Weak
(5)	France (Annuities)	1814–1922	0.908	0.206	1.75	1.34	Yes	F	F	Deterministic; No Drift	Strong
	English Nominal Wages										
(6)	Cotton-Textile Industry	1807–1913	0.990	0.399	1.42	1.28	Yes	F	F(4)	Deterministic; No Drift	Strong
(7)	Agricultural Laborer	1789–1913	0.958	0.234	1.61	1.09	No	F	F	Deterministic; No Drift	Strong
	Foreign Trade										
(8)	External Trade Turnover — England[τ]	1802–1914	0.987	0.219	1.90	1.22	Yes	F	S	Smooth Trend	Very Strong
(9)	External Trade Trurnover — France	1827–1913	0.936	0.388	1.92	3.56	Yes	S	F	Local Level With Drift	Strong
	Per Capita Coal Production and Demand										
(10)	Production — England[τ]	1855–1917	0.988	0.506	1.97	1.09	Yes	F	S(2)	Smooth Trend	Weak
(11)	Consumption — France	1827–1913	0.995	0.238	1.90	1.42	Yes	S	F	Local Level With Drift	Strong
	English Per Capita Production										
(12)	Cast Iron	1839–1914	0.918	0.403	1.99	1.48	Yes	F	F(3)	Deterministic; No Drift	Strong
(13)	Lead[τ]	1855–1920	0.991	0.319	1.93	0.94	Yes	S	F(2)	Local Level With Drift	Strong
(14)	French Private Savings Deposits	1835–1913	0.998	0.501	2.02	1.21	No	S	F(2)	Local Level With Drift	Weak
(15)	Annual Wages of French Coal Workers	1827–1913	0.980	0.236	1.78	1.31	Yes	F	S	Smooth Trend	Strong
(16)	Consumption of Mineral Fuel in France[‡]	1827–1913	0.995	0.207	1.94	1.39	Yes	F	S	Smooth Trend	Strong

UCM Trend Options: F = Fixed (Order of Slope). S = Stochastic.
[†]Index of commercial prices, expressed in terms of gold.
[τ]Log $_{10}$ transformation.
[‡]Omitted from Kondratieff (1935) and only in (1984).
Source: Appendix tables, Kondratieff (1926 [1998]); Author's calculations.

column (9) indicates 'strong' model convergence across 12 of 16 UCMs which provides further support for the model specifications because the maximum likelihood estimation — carried out by numerical optimization — was successful.

6. Comparing Long Cycle Periodicities and Endogeneity

Kuznets (1940, p. 266) criticized Schumpeter's qualitative/narrative dating methods for their inability to: (1) distinguish between endogenous and exogenous causes of long cycles, and (2) identify the primary determinants of a long cycle from an irregular movement in a time series:

> Furthermore, the distinction between cycles and irregular movements traceable to *external factors* can be made at all adequately only if successive cycles are measured and averages are struck in which the influence of external factors *can be reduced if not eliminated*. (Emphasis added)

Notwithstanding this critique, Schumpeter (1939, p. 167) understood long waves to be fundamentally driven by endogenous factors, especially the long cycle associated with the industrial revolution 'which consisted of a cluster of cycles' of which ' … the Long Wave, is completely different … that cannot be linked to a particular type of innovation … but is the result of all industrial and commercial processes of that epoch.' For both Kondratieff and Schumpeter, epoch-making technical advances (also known as 'general purpose technologies') in steam engines, electricity, and railroads were crucial drivers of capitalist long waves. However, the lack of a statistical method to determine the endogeneity of a time series hindered their empirical arguments. We apply UCMs to address both of Kuznets' criticisms below.

6.1. Comparison of Modeled Long Cycle Periodicities

Table 4 compares the published and modeled average periodicity for each of Kondratieff's sixteen econometrically modeled variables. Columns (1), (2), and (4) bring forward average periodicities from Table 2 which will be compared to the 'extracted' UCM periodicities shown in column (5).[44] Column (3) denotes the time-trend model specification; columns (6) through (8) show the difference between each modeled long cycle periodicity subtracted from the periodicity implied by Kondratieff's published turning-point years; column (9) shows each UCM-extracted measure of endogeneity or 'damping factor.' Columns (10) and (11) display the intermediate and short-cycle periodicities produced by the UCM.

The bottom row of Table 4 shows that the average modeled long cycle periodicity is bounded between 45.2 and 49.3 years, with Kondratieff's average 'Published' periodicity at 47.9 years. The average difference from this benchmark produced by the three different methods is also fairly bounded with the time-trend models averaging 2.8 years shorter (45.2 years) and the UCM-average and extracted models averaging 1.4 years shorter (46.6 years) and 1.3 years longer (49.3 years), respectively. The biggest

[44]As was done in Table 2, the average UCM periodicity shown in column (4) is the difference between successive predicted long wave peak/trough years; the 'extracted' UCM periodicity shown in column (5) is a *calculated* (scalar) value and thus a direct measure of the periodicity of a long cycle produced by the STAMP program.

Table 4. Comparison of Kondratieff periodicities (Yrs.).

Row	Column	(1) Published Average	(2) Time-trend Average *	(3) Model†	(4) Average *	(5) UCM Extracted	(6) Diff. from 'Published': Time-trnd	(7) Diff. from 'Published': UCM-Avg	(8) Diff. from 'Published': UCM-Extr.	(9) Damping Factor (ρ)	(10) UCM: Other Cycles: Intermed.	(11) UCM: Other Cycles: Short
	WPI											
(1)	England	53.3	56.0	TT3	58.5	58.9	−2.8	−5.3	−5.7	0.9998	18.3	5.1
(2)	France	47.0	41.0	TT3	58.0	60.9	6.0	−11.0	−13.9	0.9847	11.5	5.8
(3)	USA	53.0	51.8	TT3	55.8	56.1	1.3	−2.8	−3.1	1.0000	18.1	6.6
	Average	51.1	49.6		57.4	58.7	1.5	−6.3	−7.6	0.9948	16.0	5.8
	Bond Indices											
(4)	England (Consols)	52.8	52.3	TT3	55.5	58.8	0.5	−2.8	−6.0	1.0000	6.0	5.5
(5)	France (Annuities)	51.3	52.0	TT1	56.5	58.9	−0.8	−5.3	−7.7	0.9997	13.5	5.6
	Average	52.0	52.1		56.0	58.8	−0.1	−4.0	−6.8	0.9998	9.8	5.5
	English Nominal Wage											
(6)	Cotton-Textile Industry	47.8	40.8	TT3	42.0	43.6	7.0	5.8	4.1	1.0000	8.4	5.1
(7)	Agricultural Laborer	50.8	48.8	TT1	51.8	55.5	2.0	−1.0	−4.8	0.9885	16.8	6.3
	Average	49.3	44.8		46.9	49.6	4.5	2.4	−0.3	0.9942	12.6	5.7
	Foreign Trade											
(8)	External Trade Turnover — England	53.5	57.5	TT2	42.0	55.1	−4.0	11.5	−1.6	0.9998	18.3	5.1
(9)	External Trade Turnover — France	48.0	47.0	TT1	49.8	49.6	1.0	−1.8	−1.6	0.9963	8.7	3.1
	Average	50.8	52.3		45.9	52.4	−1.5	4.9	−1.6	0.9981	13.5	4.1
	Per Capita Coal Production and Demand											
(10)	Production — England	42.0	38.0	TT2	40.0	54.1	4.0	2.0	−12.1	1.0000	8.6	5.6
(11)	Consumption — France	44.0	40.0	TT3	45.5	47.4	4.0	−1.5	−3.4	1.0000	8.8	8.0
	Average	43.0	39.0		42.8	50.7	4.0	0.3	−7.7	1.0000	8.7	6.8
	English Per Capita Production											
(12)	Cast Iron	43.0	43.0	TT2	42.5	39.1	0.0	0.5	3.9	0.9844	8.8	7.2
(13)	Lead	44.0	44.0	TT3	39.5	40.6	0.0	4.5	3.4	1.0000	14.7	9.5
	Average	43.5	43.5		41.0	39.8	0.0	2.5	3.7	0.9922	11.7	8.3
(14)	French Private Savings Deposits	48.0	40.0	TT2:	44.0	43.0	8.0	4.0	5.0	0.9873	13.2	7.9
(15)	Annual Wages of French Coal Workers	46.0	40.0	TT1	38.5	41.2	6.0	7.5	4.8	1.0000	8.9	2.0
(16)	Consumption of Mineral Fuel in France ‡	n/a	42.0	TT3	44.0	47.4	n/a	n/a	n/a	1.0000	8.5	6.6
	Average	***47.9***	***45.2***		***46.6***	***49.3***	***2.8***	***1.4***	***−1.3***	***0.9958***	***11.8***	***5.8***

*Average of T-T and P-P periodicities (Table 2).
†Time-trend specification: TT1: $Y_t = \beta_0 + \beta_1 t$. TT2: $Y_t = \beta_0 + \beta_1 t + \beta_2 t^2$. TT3: $Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$. ‡$Y_t = \beta_0 + \beta_1 t + \beta_3 t^3$.
Source: Kondratieff (1926 [1998], p. 37) and Appendix tables; Author's calculations.

divergence from the published benchmark is for the UCM-extracted periodicities for the WPI for France and per capita English coal production. However, the overall results across all models and variables generally support a long cycle expectation of about fifty years.

6.2. Endogeneity

Both Kondratieff and Schumpeter's theory of long waves saw endogenous technical and organizational change as fundamental determinants of capitalist growth. These transformations took a variety of forms including the 'bunching' of inventions and innovations that initially appear during the declining phase of a long cycle. As new technologies diffuse into existing and new sectors, older production techniques become unprofitable and thus uncompetitive, giving way to more efficient firms and setting the stage for stronger aggregate growth.

Given the initial pulse to economic growth from an endogenous change, the UCM 'damping factor' can indicate the degree to which the structural parts of the long cycle either self-propagate or diminish. Goldstein (1999, p. 76) observed that a damping factor (ρ) close to zero implies that:

> if a continuum of long waves exists, this series of cycles is highly dependent on exogenous (random) shocks for its propagation. In the extreme, when $\rho_i = 0$, the cycles fades into oblivion — it ceases to exist. When ρ_i has a high value, the cycle is *self-generating, and in the extreme ($\rho_i = 1$) fully endogenous*, independent of exogenous shocks for its survival. (Emphasis added)[45]

Column (9) on Table 4 displays the damping factor for each of twelve time series analyzed by Kondratieff. Again, Goldstein (1996, p. 965) found that values of ρ less than or equal to one imply that the cycle is endogenous 'with a virtual nonexistent dependence on random shocks for propagation.'[46] The results indicate that every variable — except English cast iron production — is endogenously determined. This is an important result because it directly addresses Kondratieff's Soviet critics (including Trotsky) who claimed that long waves are determined by factors exogenous to the capitalist system. The consistent estimates of long cycle periodicities and measures of endogeneity across a range of price, output, and trade data underscore their primacy as valid and reliable measures of long waves. Moreover, Goldstein (1996, 1999) found that these estimates are consistent with long wave periodicities for real per capita GDP across the major advanced economies as well as tests for endogeneity of the postwar USA business cycle.

6.3. UCM Intermediate and Short Cycle Periodicities

The STAMP software program can calculate the periodicity not only for the long cycle, but also for short and intermediate cycles as well. Schumpeter's 3-cycle trigonometric model predicts an intermediate and short cycle lengths of 9.5 and 3.5 years, respectively. Kondratieff (1935, p. 105) did not estimate periodicities for shorter cycles but acknowledged Kitchin and Juglar's estimates for the short and intermediate cycle:

[45]Please see the Technical Appendix, section A 2.3.2 for further details.
[46]STAMP does not estimate significance tests for ρ, however, tests using STATA indicated that these values are statistically significant.

> When in economics we speak of cycles, we generally mean seven to eleven year business cycles. But these seven to eleven year movements are obviously not the only type of economic cycles. The dynamics of economic life is in reality more complicated. In addition to the above-mentioned cycles, which we shall agree to call 'intermediate', the existence of still shorter waves of about three and one-half years' length has recently been shown to be probable.

Columns (10) and (11) show the extracted UCM periodicities for the intermediate and short cycles for each of Kondratieff's twelve modeled variables. The results are fairly consistent across time series with the intermediate and short cycle periodicities averaging 11.8 and 5.8 years, respectively. The intermediate cycle ranged from 6.0 years for the English bond index to 18.3 years for the English WPI. French coal worker wages had the lowest short-cycle periodicity of 2.0 years and English lead production had the longest at 9.5 years. Taken together, these estimates are broadly consistent with Schumpeter's theoretical expectations and empirical estimates of long and shorter-term business cycles.

6.4. Other Methods

6.4.1. Spectral Analysis

Goldstein (1988, p. 66) provides a cogent description of spectral analysis that uses 'sophisticated routines to search for regular fixed periodicities' unlike irregular periods produced by moving averages. In the following footnote he further elaborates how:

> Spectral analysis creates a function in which the degree of correlation to a sine wave to the time series data is expressed in relation to the wavelength of the sinewave. Cross-spectral analysis concerns the correlation among more than one series conceived as sine waves. Fourier analysis breaks a time series down into a set of sine waves of different wavelengths whose sum best approximates the series.

To that extent, modern spectral analysis has developed complementary tools to analyze both the periodicity and frequency of the long cycle. While both the time and frequency domain contain the same raw information, spectral analysis borrows from the mathematics of Fourier transformations for extracting deterministic trends using what Renderos (2014) terms the 'Significant Pass Filter (SPF).' In a nutshell, this technique estimates the deterministic component from all the low and high spectrum, without imposing band-pass restrictions. Therefore, our turning-point years will be generated using this technique.[47]

van Ewijk (1982), applied spectral analysis to price and production time series from the late 19th century to 1930 for Britain, France, Germany, and the USA. Although he acknowledges the 'n = 2.5' long wave sample problem,[48] Van Ewijk's periodograms across these four economies confirmed a long wave mostly in the price data through 1930, but could not confirm its presence through 1977 due to non-stationarity in the time series. No precise dating of long wave peaks and troughs was provided for any pricing or production data. Later work by Grenier (1984) applied spectral techniques

[47]See Renderos (2014), especially pp. 13–18. Details for the EViews spectral model diagnostics that produced the turning points for each time series are available from the author.

[48]van Ewijk (1982, p. 476): 'For spectral analysis, series of about three long cycles should be considered to be the absolutely minimum requirement.'

Table 5. Preindustrial long cycle peaks (P), toughs (T) and implied periodicities

Author of Study	Year	Countries	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*		Average Periodicities: Peak-to Peak	Average Periodicities: Trough-to Trough	Average Periodicities: Overall
Imbert	1959	England		1480	1507	1530	1540	1558	1571	1597	1620	1649	1688	1710	1732				46.0	45.0	45.5
		France			1510	1530	1539	1595			1612	1650	1671	1712	1733				63.5	50.0	56.8
		Germany			1505	1535	1546	1570	1580	1590	1605	1620	1670	1700	1730				41.3	46.0	43.6
		Spain			1510	1530	1540	1562	1594	1601	1618								35.5	36.0	35.8
Braudel	1972	Europe	1460	1483	1509	1529	1539	1559	1575	1595	1621	1650							41.8	40.3	41.0
Frank	1978	England										1670	1689	1720	1747	1762	1790		46.0	50.5	48.3
Baehrel	1961	France							1573	1594	1625	1655	1689	1725	1764	1785			63.7	63.7	63.7
Wageman	1931	World											1690	1720	1730	1763	1790		43.0	50.0	46.5
Mauro	1964	World								1590	1620	1640	1660	1720	1730	1775	1792		61.7	57.3	59.5
Metz	1983	Germany		1494	1518	1534	1569	1591			1613	1636	1680	1703	1724	1762	1780		55.8	54.5	55.1
Goldstein	1988	Composite		1495	1509	1529	1539	1559	1575	1595	1621	1650	1689	1720	1747	1762	1790		44.5	46.8	45.7
Average																		**Avg.**	49.3	49.1	49.2
Stdev																		**Stdev.**	10.0	7.7	8.5

Source: Goldstein (1988, pp. 67 and 72–74).

to French prices from 1500 to 1790 but did find evidence of a long wave. Kuczynski (1978) analyzed world production and innovation data from 1850 to 1976, but could only weakly confirm a long cycle. Metz and Stier (1992) applied filter design techniques to analyze simulations of long waves. Although only a few data series are examined, an oscillating long-term trend for UK coal production can be discerned from 1700 to 1820 (pp. 76–77). Metz (2011) presents a cogent and comprehensive review of the major quantitative techniques used to estimate long wave periodicities including the classical decomposition approach (see Appendix 1), ARIMA models and spectral analysis, and the structural time series (STS) models such as the UCM used in this paper. Metz identifies long waves in British pig iron production and UK per capita GDP using Maddison's long-term dataset.

6.4.2. Wavlet Models

Gallegati et al. (2017, pp. 131–133) recently applied 'wavelet' analysis to extract Kondratieff long wave periodicities for price indices in France, Great Britain and the US from 1791 to 2012. This approach expands upon methods and techniques drawn from spectral analysis (e.g., band-pass filters, transfer functions), that can incorporate shorter-term structural breaks to produce long waves with varying periodicities. Wavelet analysis also shares some features with UCMs such as not requiring stationarity in a time series.[49] This study also recognized a time series 'as an overlay of many cyclical components of different lengths and frequencies.' Although wavelet analysis does not provide a measure of endogeneity, it is able to provide nonparametric estimates of long cycle periodicities. More importantly, Wavelet models of wholesale prices generated peak and trough years of long cycles very similar to those identified in Kondratieff.[50]

7. Chronology of Long Wave Price Cycles

A central debate in the literature has focused on the chronology of Kondratieff's long cycles in commodity prices. The fact that his longest time series data for England (1780–1925) and the USA (1780–1925) covered only two complete cycles was a primary source of criticism. Even his less polemical colleague and critic, S.A. Pervushin noted that the 'statistical record' neither supported Kondratieff's estimate of fifty-year cycles nor Oparin's refutation of their existence 'since they were working with [only] two-and-a-half cycles' (1926 [1998], p. 109). Kondratieff was quite aware of the '$n = 2.5$' problem and recent archival work by Mustafin (2019) indicates that Kondratieff was actively engaged in compiling time series that extended back to earlier centuries. In fact, Mustafin uncovered 'rough drafts' of Kondratieff's analysis and data used to estimate long wave periodicities based upon Danish grain prices from 1600 to 1902 such that '… these documents indicate that the problem of the existence of long cycles before the Industrial Revolution had attracted the interest of Kondratiev' (2019, p. 5). Furthermore,

[49]Gallegati et al. (2017, p. 133): 'Wavelets provide an analytic structure which can be used to provide relatively efficient estimations of variation of frequency, by time and by scale. In short, the advantages of wavelet analysis are preeminent in the initial analysis of economic and financial data.'

[50]Average long wave periodicities for English prices based on Kondratieff (1935), UCMs, and Gallegati et al. (2017), are: 58.3. 58.5, and 58.3 years, respectively; for the US, 53.0, 55.8, and 50.5 years, respectively; and for France, 47.0, 58.0, and 58.0 years, respectively.

Oparin analyzed and confirmed a 'horizontal trend' in Kondratieff's graph of English wheat prices during the 17th and 18th centuries (Kondratieff 1926 [1998], p. 174).[51]

7.1. Preindustrial Long Waves

Goldstein (1988) presents peak and trough years from several major studies of preindustrial prices based upon 'qualitative data and price histories … [that] demonstrate surprising consensus.' Goldstein culled through the major studies of 15th, 16th, and 17th century prices by Wagemann (1931), Imbert (1956), Baehrel (1961), Mauro (1964), Braudel (1972), Frank (1978), and Metz (1983). Table 5 identifies the peak and trough years associated with each of these studies that cover approximately four complete long cycles as well as the implied average periodicities based on peak-to-peak and trough-to-trough averages. The table demonstrates a high degree of conformity in the individual turning point years as well reasonably bounded averages and standard deviations.

7.2. Additional Evidence of Preindustrial Price Cycles

Applying our earlier comparative modeling scheme used to evaluate Kondratieff's published turning points for the 18th, 19th, and 20th centuries, we model turning point years for the English BOE WPI data (1660–2016) as well as Kirkland (1917) data on English wheat prices (1600–1917); however, the modeled estimation periods will be truncated at 1790. The top portion of Table 6 compares the long cycle turning point years and average periodicities from Goldstein (1988) as well as those based upon (1) smoothed 9-year residuals for a second-order polynomial econometric time-trend model (T-T); (2) an unobserved components model (UCM), and (3) a spectral analysis model using the EViews 13.0 software package.[52] For the BOE WPI time series, the correspondence in predicted turning point years is more consistent among the three models compared to Goldstein's dates. Nevertheless, the average periodicity is bounded between 45.7 and 58.0 years and is consistent with Thompson (1992, p. 38) who notes that for the leading economic sectors ' … there is an unusually high degree of consensus in the literature on the timing of pre-nineteenth century long waves.' Louca (1999) concurred with Thomson, noting the close correspondence in long wave chronologies between disparate analysts such as Trotsky and Van Gelderen:

> The coincidence of so many authors on the chronology [of long waves], although working independently, suggests the distinctive features of the historical development of nineteenth century capitalism. (Louca 1999, p. 8, footnote 7)

Figure 5 displays the logarithm of the observed WPI time series as well as the modeled values. The turning point years are quite similar across models, although they only cover one and a half long cycles from 1660–1790. The biggest divergence occurred between Goldstein's first long cycle peak (1650) and the time-trend model (1670), followed by

[51]Oparin took the data from Kirkland (1917) which was reproduced in Kondratieff (1926 [1998], pp. 227–231).

[52]Available as a downloadable EViews 'add-in' that incorporates the latest filtering techniques including band-pass filters developed by Baxter and King (1999) and Chistiano and Fitzgerald (2003); See Renderos (2014) for download instructions and technical details of the EViews spectral analysis program.

Table 6. Modeled preindustrial long wave price cycles: peaks (P), toughs (T) and implied periodicities.

	WPI-BOE (1661–1790)															Average Periodicities		
	P	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*		Peak-to Peak	Trough-to Trough	Overall
Goldstein Prices(1988)	1495	1509	1529	1539	1559	1575	1595	1621	1650	1689	1720	1747	1762	1790		44.5	46.8	45.7
Polynomial T-T (2nd-degree)									1670	1689	1704	1740	1766	1790		48.0	50.5	49.3
UCM									1667	1686	1710	1742	1760	1790		46.5	52.0	49.3
Spectral Analysis									1661	1685	1706	1733	1788	1790		63.5	52.5	58.0
															Avg.	50.6	50.5	50.5
															Stdev.	8.7	2.6	5.3
	English Wheat Prices (1600–1790)																	
	P	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*	*P*	*T*				
Goldstein Prices(1988)	1495	1509	1529	1539	1559	1575	1595	1621	1650	1689	1720	1747	1762	1790		44.5	46.8	45.7
Polynomial T-T (3rd-degree)								1612	1638	1645	1653	1692	1700	1750		31.0	46.0	38.5
UCM								1622	1631	1648	1670	1687	1695	1718		32.0	32.0	32.0
Spectral Analysis								1622	1637	1641	1661	1689	1710	1744		36.5	40.7	38.6
															Avg.	36.0	41.4	38.7
															Stdev.	6.2	6.8	5.6

Source: Goldstein (1988, p. 67) and Author's calculations.

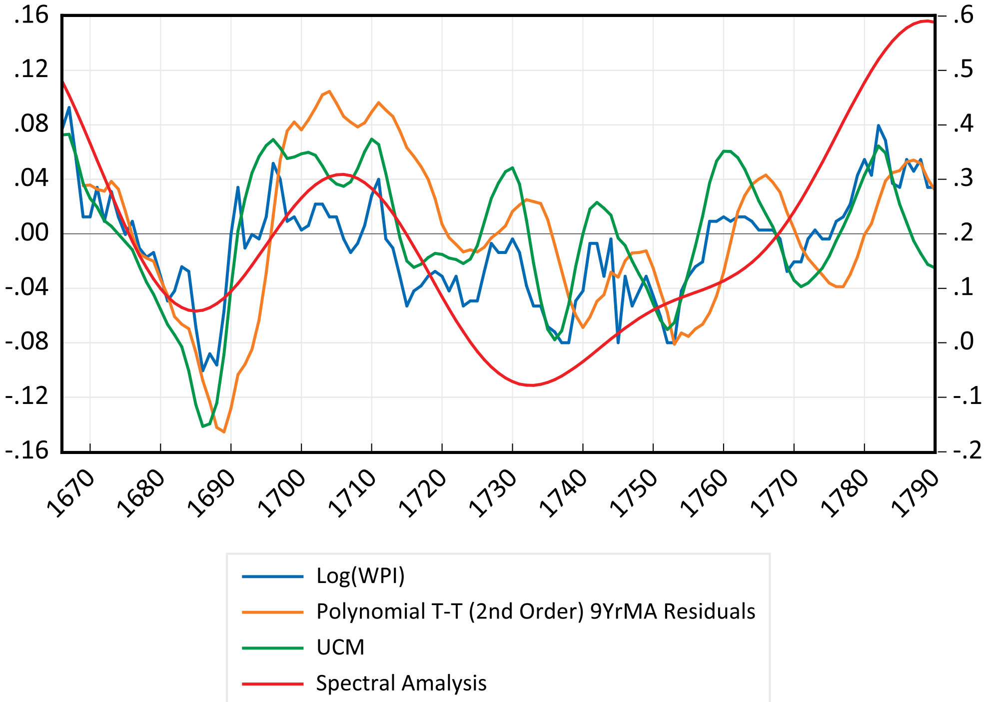

Figure 5. WPI for the UK, 1661–1790: actual vs. modeled. Source: Kondratieff (1998) and author's calculations.

the UCM (1666) and Spectral Model (1661); however, he following trough (1689) was clustered within four years of all the turning points. Concern has been expressed about Kondratieff's first-cycle tough in 1790. Figure 5 shows that while the UCM estimates a clear trough in that year, the spectral analysis model only shows a very marginal decline from the peak in 1788.

On the other hand, none of the estimated models provided empirical support for a 50-year long cycle in English wheat prices. This is likely due to the notable volatility in the observed time series which generated a UCM long cycle (scalar) estimate of 16.1 years.[53] Figure 6 displays the logarithm of the observed prices as well as the modeled values. Furthermore, as Oparin (1926 [1998], p. 75) observed that: 'Despite a number of differences between the movement of the price of wheat and the movement of the general level of prices, we can state that the secular trend in the movement of prices is a horizontal line at approximately the level given [in Figure 15, p. 174] by Professor Kondratiev.' Oparin's comment is also consistent with Thompson (1992, p. 25) who observed that: 'The pre-nineteenth century series is characterized by more frequent fluctuations than the post-nineteenth century data — an outcome attributed to varying patterns of warfare and changes in the nature of the leading sector.'

Indeed, the bottom of Table 6 shows that the model peak and trough years are almost all below Goldstein's respective peaks and troughs. This pattern is likely due to

[53]Full details of the UCM model diagnostics are available from the author.

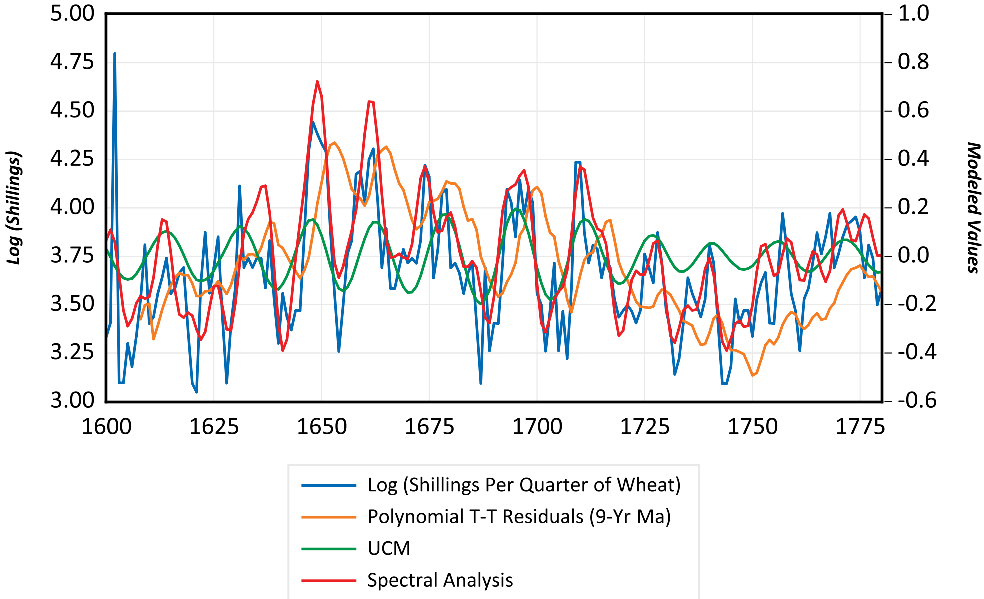

Figure 6. English wheat prices per quarter bushel, 1600–1790: actual vs. modeled. Source: Kondratieff (1998) and author's calculations.

Goldstein's dating year scheme is based on a composite price series from several studies. Another problem is that there is a notable 'sawtooth' pattern (equivalent to a pattern of negative serial correlation) in the observed time series while the gap between Goldstein's and the models' peak and trough generally gets wider through 1790. Thus, it is not surprising that the average modeled periodicity for English wheat ranges between 31.0 and 46.0 years, which is well below Goldstein's estimates.

8. Conclusions

The reexamination of long wave cycles identified in the works of Kondratieff and Schumpeter supports their finding of a long wave periodicity of about 50 years. Applying the UCM methodology to extract long cycles from Kondratieff's original data for 16 different time series generated an average periodicity of 49.3 years. Using Kondratieff's published turning point years as well his re-estimated time-trend models, yielded an average long cycle periodicity of 45.5 years. Heretofore unpublished significance tests, goodness-of-fit measures and other diagnostic information regarding Kondratieff's econometric models are provided.

Secondly, the chronology of preindustrial turning point years based on the leading economic historical scholarship of this era supported a long wave periodicity of about fifty years.[54] New archival evidence from Kondratieff's unpublished papers indicate

[54]Louca (1999, p. 8, fn. 6) who concurs with Thompson (1992), and notes how Trotsky's chronology of long waves '… corresponds closely to the chronologies of by previous authors, namely the Italians or Van Gelderen, probably unknown to Trotsky. The coincidence of so many authors on the chronology [of long waves], although working independently, suggests the distinctive features of the historical development of nineteenth century capitalism.'

that he was actively engaged in compiling new data on preindustrial prices to extend his long wave research into earlier centuries. Such a finding is consistent with his published graphical presentation of English wheat prices from 1600 through 1917. New long cycle periodicities for the English wholesale price index (WPI) and English wheat prices were estimated using polynomial time-trends, UCMs and spectral analysis models. While all three techniques produced roughly similar turning-point peak years and implied periodicities of about fifty years for the English WPI time series, this was not true for the highly volatile preindustrial English wheat prices which exhibited cycles between 16 and 40 years.

A third conclusion is that Schumpeter's periodicities derived from a trigonometric specification of the short, intermediate, and long cycle were most consistent with periodicities generated by the UCMs; the average periodicity for the short and intermediate cycles are 5.8 and 11.8 years, respectively. The UCM approach (as well as recent wavelet models) provides a valid and reliable framework for estimating long wave periodicities based upon rigorous but flexible trend-cycle decomposition methods. In particular, the flexible level and slope UCM specifications directly address Solomou's (1998, p. xiv) claim that 'trends are best depicted as stochastic rather than deterministic processes,' Although this is true for some time series, our estimates found a mix of deterministic and stochastic trend specifications which is also consistent with similar specifications found in Goldstein (1999). Long cycles are present in both Kondratieff's nominal and output measures. The latter results are robust across methods and estimation periods where the average long cycle periodicity was between forty and fifty years for coal, cast iron, and lead production in England and France. These results are consistent with those published in Gallegati et al. (2017) and Kriedel (2006) who applied non-parametric wavelet techniques to model nominal prices and net national product for the leading European economies and Russia, respectively. Both approaches confirmed specific years of long cycle turning points similar to those published by Kondratieff.

A fourth result is consistent and significant evidence of endogenously propagated long cycles in all of Kondratieff's time series. These results support Tylecote's (1998, p. xxx) elaboration of Kondratieff's theory of capitalist endogenous growth that accounts for so-called extra-economic factors such as 'the concentration of capital, the accumulation of technical inventions, and the determination of "upheavals".' The last factor has posed a significant challenge to long wave theory because critics assumed that wartime inflation is always the *result* of widespread shortages as productive resources are diverted toward military activities. Thus, critics have claimed that long cycles in prices and interest rates were simply statistical 'artifacts' of fundamentally exogenous shocks to a capitalist economy. However, the estimated UCM damping factors consistently reject exogenous explanations for long cycles and support Tylecote's observations that Kondratieff needed to present a more integrated and endogenous dynamic between technical change, capital costs and real investment that generate political and social frictions and upheavals. For example, during the descending phase of a long cycle, falling real and nominal interest rates provide the catalyst for long-term debt-financed infrastructure investments (e.g., canals). Thus, there will be a delay in manufacturing innovation, investment and output because sales must begin to rise *before* internal financing is sufficient to fund new capital expenditures. The slow diffusion, adaption and modification of new production techniques into enterprises and industries (e.g., English steam locomotives) will further retard aggregate growth.[55] In this way, if we assume that infrastructure investment as well as inventive activity precedes increases in private capital formation, macroeconomic growth will occur *later* in the downswing and delay the foundation for the following long upswing.

Tylecote also reconciled the apparent contradiction between Kondratieff's assumption that technical innovations are fundamentally a counter-cyclical phenomena that could not be positively associated with price increases (1926 [1998], p. xxxiii). Had he lived, Kondratieff could have responded by distinguishing between technological changes that Schumpeter (1934) identified as cost-reducing 'process innovation' — that typically occur during a long cycle downswing — versus a 'product innovation' which raise the demand and price for a product during a long cycle upswing. The empirical evidence presented in this study further support both Kondratieff and Schumpeter's contention that economic and financial long cycles of approximately fifty years are fundamentally endogenous to capitalist economies.

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[55] Kriedel (2006) found that the diffusion of 19th century European railway networks followed a logistical S-curve pattern whose turning points *preceded* those identified in Kondratieff's second long wave. This result supports Schumpeter's theory of endogenous technical changes embodied in general purpose technologies (e.g., steam locomotives and railway networks) that drive long economic cycles.

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Appendices

Appendix 1. Classical Time Series Decomposition

The mechanical decomposition of a time series into its trend, cycle, seasonal, and irregular component was developed by Mitchell (1927); and Burns and Mitchell (1946). Their multiplicative specification is defined as Y_t = Trend (T) X Cycle(C) X Seasonal (S) X Irregular (I) where Y_t is the actual value of a time series and the trend values for 'T' are the predicted values from a linear regression time-trend model of the centered moving average (CMA) of Y_t. The cyclical value, 'C' is defined as the ratio of the CMA/T. Obviously, for annual data there would be no seasonal 'S' values, but for quarterly and monthly data, they would also be centered mid-year moving averages. The irregular component is a residual value that would typically have a default historical value of 1.00 but could take on different *ex ante* values for a forecast. Burns and Mitchell assumed that T, C, and S would be relatively stable with the irregular factor, I capturing any non-recurring (i.e., random) error in the actual time series. Each of these components was assumed to be highly stable, however, subsequent advances in time series econometrics — especially the UCM — challenged this idea.

Appendix 2. Technical Overview of the UCM

In order to grasp the basic logic and versatility of the UCM, assume a simple exponential time-trend regression model, specified as ln $(Y_t) = \alpha + \beta$ Time $+ \varepsilon_t$, where α and β are parameters that will be estimated using ordinary least squares and ε_t is a random and normally distributed error term. In this formulation, the y-intercept (α) and slope (β) are estimates of fixed parameters that assume a constant rate of growth. Now the essence of any economic or financial cycle is that there are recurring patterns peaks and troughs that identify changes in the *direction* of growth rates or levels over a complete cycle. However, if α and β were allowed to *evolve* over time — either stochastically or deterministically — it is possible to obtain more precise estimates of the trend and cyclical components embedded in long waves. To do this, the *change* in each time-trend parameter can be defined as $\Delta\alpha \equiv \alpha_t - \alpha_{t-1} = \eta_t$ and $\Delta\beta \equiv \beta_t - \beta_{t-1} = \zeta_t$ where both error terms (η_t and ζ_t) are assumed to be normally and identically distributed random variables with a mean of zero and constant variance such that $\eta_t \sim \text{NID}(0,{\sigma_\eta}^2)$ and $\zeta_t \sim \text{NID}(0,{\sigma_\zeta}^2)$.[56]

For example, in Kondratieff's model of English coal demand, the first and second-order regression coefficients implied that coal demand grew by about 1.7 percent per year.[57] However, a UCM allows these parameters to evolve, depending on the variance in the y-intercept and slope error term, ${\sigma_\eta}^2$ and ${\sigma_\zeta}^2$. More importantly, the UCM can extract the periodicities of the short, intermediate, and long wave cycle by incorporating both time-varying structural (e.g., α and β) and stochastic (${\sigma_\eta}^2$ and ${\sigma_\zeta}^2$) parameters. This approach avoids arbitrary and potentially biased estimates of cycle lengths, but also produces measures of cyclical endogeneity ('damping factors') which were an especially important concern of Kuznets and Soviet critics of Kondratieff's statistical methodology.

A.2.1. The UCM Model Specification

A UCM is an econometric model with time-varying parameters including a trend, cycle, seasonal, and irregular component; explanatory or independent variables, dummy variables and autoregressive terms can also be incorporated into the model. Our analysis of Kondratieff's twelve time series employs a univariate UCM defined as:

$$y_t = \mu_t + \psi_t + \varepsilon_t \text{ and } \varepsilon_t \text{ is NID}(0,\ \sigma_\varepsilon^2) \quad (A1)$$

where y_t is the dependent variable, μ_t is the trend component, ψ_t is the cyclical components, and ε_t is a random (or 'irregular') error term which must be estimated using the STAMP software

[56]All estimations are carried out using OxMetrics7 'Structural Time Series Analyzer, Modeler, and Predictor' STAMP Version 8.2 software program developed by Koopman et al. (2009).

[57]The model is estimated in semi-log form such that adding the anti-logs of the slope coefficients provides the average rate of change in tons of coal per 1,000 inhabitants (see table 1, row (10), EViews 10 slope estimates).

package. The program's output includes estimates of the periodicity for the short, intermediate, and long cycle as well as year-by-year predicted values for each cycle. In this way, the UCM draws upon elements in Schumpeter's three-cycle trigonometric formulations while allowing for flexible trend and cycle specifications.[58] This approach also provides a more general formulation of the classical univariate time series decomposition methodology devised by Mitchell (1927) and Burns and Mitchell (1946) as well as the modern Box-Jenkins ARIMA model.[59] In contrast to these earlier methods, the UCM allows the trend and cycle components to evolve over time and does not require a stationary time series or the loss of degrees-of-freedom due to the smoothing of residuals. The UCM is also superior to the Hodrick-Prescott low-frequency filter because it uses bandpass filters to extract business cycles that occur at both low and high frequencies.

A.2.2. Modeling the UCM Trend Component

A.2.2.1. Random Walk Without Drift. The UCM trend component, μ_t can be modeled either as a stochastic or fixed parameter. If a time series exhibits a cyclical pattern with little or no trend, then μ_t can be specified as a 'random walk without drift' such that:

$$\mu_t = \mu_{t-1} + \eta_t \text{where the error term, } \eta_t \text{is NID } (0,\ \sigma^2_\eta) \tag{A2}$$

A more restrictive version of equation A2 would be for a 'mean reverting' time series where $\sigma^2_\eta = 0$ and thus $\mu_t = \mu_{t-1}$. Such a model would be equivalent to an ARIMA (0,1,1) specification, where the first-difference in the dependent variable, y_t is simply a function of an MA(1) term. Strong mean reversion would be ensured if the variance of the trend error term were set equal to zero ($\sigma^2_\eta = 0$).

A.2.2.2. Local Linear Trend Model. A UCM where both the level and slope of a trend parameter changes over time is termed a 'locally linear trend' (LLT) model and can be defined by a separate level and slope equation:

$$\text{Level:}\quad \mu_t = \mu_{t-1} + \beta_t + \eta_t \text{where the error term, } \eta_t \text{is } '\text{niid}'(0,\ \sigma^2_\eta) \tag{A3}$$

$$\text{Slope:}\quad \beta_t = \beta_{t-1} + \zeta_t \text{where the error term, } \zeta_t \text{ is } '\text{niid}'(0,\ \sigma^2_\zeta) \tag{A4}$$

In equation A3, μ_t is the stochastic (random) level of the trend parameter that depends on its prior value as well as a slope parameter (β_t) and error term. Equation A4 defines the stochastic slope parameter, β_t which varies with its prior value and an error term. The error terms in both models are assumed to be uncorrelated. A UCM specified with stochastic level and trend parameters would be appropriate for modeling a time series with a curved or non-linear slope. This specification is also equivalent to an ARIMA (0.2.2) where the second-difference of Y_t is a function of an MA(1) and MA(2) term.

The above recursion can be restricted for modeling a time series that randomly meanders around a linear trend. For example, a UCM with a fixed level ($\sigma^2_\eta = 0$) but stochastic trend is described as a 'random walk with fixed drift.' This specification was used to model Kondratieff's English external trade turnover, English coal production, annual wages of French coal workers and French consumption of mineral fuel.

Restricting both error variances to be equal to zero ($\sigma^2_\eta = \sigma^2_\zeta = 0$) creates a 'deterministic' linear time-trend model.[60] This trend specification was used to model Kondratieff's WPI for England and

[58]In his evaluation of Kondratieff's statistical methodology, Solomou (1998. p. lxiv) claims that: 'Fitting a deterministic time-trend line to the data assumes that there exists a trend-stationary growth path in the relevant time series. This assumption is unlikely to hold over long periods. There is much evidence that trends are best depicted as stochastic rather than deterministic processes.' The UCM results presented in this study suggest that both types of specifications are appropriate for different time series.

[59]Box and Jenkins (1970) developed the autoregressive integrated moving average (ARIMA) model which assumed that the trend component of any time series is a fixed value which can be removed by differencing.

[60]ARIMA first-difference tests for stationarity typically fail to correctly identify smooth stochastic trends which results in biased estimates when fitting models to detrended series. Gallegati et al. (2017) observed that econometric tests of long waves based on first-differences are suspect because of the arbitrary method to eliminate trend.

the USA, the French bond index, nominal wages for English cotton-textile and agricultural workers and English production of cast iron. BOE variables that used this specification included English Consols.

A.2.3. Modeling the UCM Cycle Component

A.2.3.1. A Deterministic Cycle Specification. Similar to the trend component model, the cyclical component, ψ_t can be modeled either as a deterministic or stochastic trigonometric cycle (or cycles). The deterministic cycle is characterized by more restrictive model parameters in the following specification where the frequency of the cycle, λ is bounded between 0 and π such that:

$$\psi_t = \alpha \cos \lambda_t + \beta \sin \lambda_t \qquad (A5)$$

Assuming that time, t is observed for each period, the cycle component, ψ_t has an approximate periodicity of $2\pi/\lambda$.[61] In addition, Fourier analysis demonstrates that complex cyclical patterns can be written as a sum of a finite series of sinusoidal functions similar to that shown in equation A5. The STAMP econometric software program allows the user to specify a short, intermediate, and long cycle to capture different dynamics across cycles.

A.2.3.2. A Stochastic Cycle Specification. A less restrictive formulation would allow all the parameters in equation A5 to vary to create a stochastic cyclical model defined as:

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda \sin \lambda \\ -\sin \lambda \cos \lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \nu_t \\ \nu_t^* \end{bmatrix}, t = 1 \ldots, T \qquad (A6)$$

where ρ is commonly known as the ‘damping factor’ which must lie between 0 and 1; λ is the frequency in radians which is bounded between 0 and π; and ν_t and $\nu_t{}^*$ are uncorrelated error terms with zero mean and common variance $\sigma^2{}_\nu$. Model (A6) is designed to capture a wide range of patterns of time series without introducing an unwieldy number of parameters. The reduced form of (A6) permits the cycle to be separated into a deterministic and stochastic component:

$$Y_t - (2\rho \cos \ \lambda) Y_{t-1} + \rho^2 Y_{t-2} = \nu_t - (\rho \cos \lambda)\nu_{t-1} + (\rho \sin \lambda)\nu*_{t-1} \qquad (A7)$$

where Y_t is the actual value and the left-hand side of the equation is the deterministic component while the right-hand side is the stochastic component. In this formulation of the cycle, the parameters λ and ρ determine the contours of the cycle which is less restrictive than the ARIMA unit-root approach.

[61]The exact periodicity is defined as $2\pi j/k$ for some integers j and k.