Understanding Fischer Black

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In the outpouring of appreciation that followed Fischer Black's untimely death on August 30, 1995, there was near unanimity that the Black-Scholes option pricing formula is the central gem around which the other smaller stones of Black's opus should be arranged.[1] This judgment was based, one gathers, on the fact that the options pricing formula is unquestionably one of the central foundation stones of modern finance theory and practice. People want to give credit where credit is due.

Unfortunately, the effect of viewing the man through the lens of the subsequent development of the field is a picture of him that is incomplete and distorted. To do justice to Fischer Black and his thought, we need to approach him on his own terms and in the context of his own time, rather than on our own terms and in retrospect. Understanding Fischer Black, we come also to understand something more about the development of the field he did so much to foster, since we can view that development through his eyes and from his perspective.

Exploring General Equilibrium

The central theme of Black's work is his general equilibrium version of the capital asset pricing model (CAPM). Published in 1972 as "Equilibrium in the Creation of Investment Goods Under Uncertainty", Black's first sketch of a general equilibrium CAPM was essentially complete already in September 1969 in unpublished form as Financial Note No. 8 [FN 8].[2] From this initial paper to "The ABCs of Business Cycles" (1981), to his first book Business Cycles

[1]See for example Merton and Scholes (1995), Figlewski (1995), and Journal of Portfolio Management (1996). See also the appreciation that followed the 1997 Nobel Prize to Robert Merton and Myron Scholes, for example Duffie (1998) and Schaefer (1998).

[2]Fischer Black Papers, Manuscript Collection #505, MIT Archives.

and Equilibrium (1987), to his second posthumous book Exploring General Equilibrium (1995a), there is a continuous thread that runs through and unites all the rest of Black's work.

The standard CAPM starts from the idea that there is an exogenously given supply of various different securities, and proceeds to work out a theory of market-clearing security prices given the risk preferences of wealth holders (Treynor 1961, Sharpe 1964, Lintner 1965). In equilibrium everyone holds the market portfolio and adjusts risk exposure by borrowing or lending at the riskfree rate. What Black did was to conceive of the assets not as financial assets but as real assets, which moreover arise endogenously because someone chooses to make a real investment. In equilibrium, both quantity and price are determined, hence it is a model of general equilibrium not partial security price equilibrium. Even so, the model inherits the essential feature of CAPM that the prices of all capital goods derive from just two fundamental prices of capital, the rate of interest and the price of risk. These two prices, and two prices only, are sufficient statistics that summarize production opportunities and investor preferences throughout the entire economy.

There were problems with Black's first general equilibrium version of CAPM--instantaneous production, single consumption good--and Black would tackle them in later work. The important point is that the essence of his later business cycle and growth theory is already implicit in the earliest work. Firms make real investments, each combining exposure to market risk and independent risk, guided by the risk tolerance of the community as reflected in the two prices of capital. The consequence is that communities with high risk tolerance get high expected excess returns, which means high growth over time, but at the cost of tolerating greater aggregate fluctuation. Communities with low risk tolerance get low growth but greater stability.

The details of Black's mature model differ, but the underlying CAPM-inspired vision of the nature of cycles and growth remains.

Also present from the start was the essential idea of Black's monetary theory. In CAPM the rate of interest is determined by borrowing and lending between aggressive and timid investors, so there is no room for monetary authorities to choose the rate as a policy variable. Moreover, in Black's version of CAPM, people maintain a constant leverage ratio appropriate to their own risk tolerance, and this implies that the total amount of borrowing and lending changes over time. When the market goes up, the timid rebalance by lending more and the aggressive rebalance by borrowing more, so credit expands. When the market goes down, rebalancing leads to credit contraction. To the extent that credit expansion and contraction take place within the banking system, there will be expansion and contraction of bank deposits and hence fluctuations of the measured money supply. Such fluctuations are evidently an effect not a cause of stock market fluctuation. In CAPM equilibrium, the money supply must be endogenous, so there is no room for monetary authorities to choose the money supply as a policy variable.

From the first, Black thought of his general equilibrium version of CAPM as an extension to the case of uncertainty of the work Irving Fisher had earlier done for the case of certainty (Black 1972b, 249). Like Irving Fisher in his 1907 The Rate of Interest, Black built his first general equilibrium model as the simultaneous solution to the household's problem of "Individual Investment and Consumption under Uncertainty" (1988b [FN 6, 1969]) and the firm's problem of "Corporate Investment Decisions" (Treynor-Black 1976 [FN 2, 1967]). Like Fisher, Black went on to build a theory of money (1970) and business cycles (1981, 1987) on these earlier foundations. Black's account of how a gold standard might be used to stabilize prices (1987, Ch. 11) makes explicit reference to Irving Fisher's similar plan (Fisher 1920).

Even Black's proposal for a new system of accounting (1993c) can be read in part as an extension to the case of uncertainty of Irving Fisher's treatise on accounting The Nature of Capital and Income (1906).

Most significant, Black seems to have taken his overall vision of the economy from Fisher. Fisher's accounting system presents a unified picture of the economy as a stock of wealth moving through time, throwing off a flow of services as it goes. In Fisher's formulation all wealth is capital, not just machines and buildings, but also land and even human beings. Indeed for Fisher human beings are the most important form of capital because the most versatile. Thus, at the highest level of abstraction, there is no distinction between the traditional categories of labor, capital, and land. All produce a stream of income (services) so all are capital, and their income discounted back to the present is their capital value. Similarly, at the highest level of abstraction, there is no distinction between the traditional categories of wages, profit, and rent. All are incomes thrown off by capital, hence all are forms of the more general category of interest, which is the rate at which income flows from wealth.

All this is in Black from the very beginning and throughout his career. The significant state variable in all his macroeconomic work is the value of wealth, understood to include both physical and human capital. Further, human capital is quantitatively the more important, although measurement problems often prevent us from seeing this clearly in economic data. As in Fisher, Black's emphasis is on the market value of wealth calculated as the expected present value of future income flows, rather than on the quantity of wealth calculated as the historical accumulation of savings minus depreciation. This allows Black to treat knowledge and technology as forms of capital, since their expected effects are included when we measure capital

at market value. As he says: "more effective capital is more capital" (1995a, 35). Also as in Fisher, capital grows over time without any restriction from fixed factors.

Black's basic model of the world has the growth rate of composite capital (physical plus human, both measured at market price) depending on the difference between a stochastic rate of return r_t and a constant consumption rate γ, so $K_{t+1} = (1+r_t- \gamma)K_t$ (1995, eqs. 4.1-4.3). This makes growth a non-stationary geometric random walk. As in Fisher, growth depends on patience which keeps the rate of consumption below the expected return on capital. But growth also depends on risk tolerance since higher tolerance means a higher expected return, and it also depends on luck since the actual rate of return may be higher or lower than expected. In this way, extending Fisher to the case of uncertainty involves more than just adding a cloud of stochastic fluctuation around the case of certainty.

To treat the case of uncertainty in a fundamental way, Black eventually departed from Fisher by generalizing in two dimensions: sectoral disaggregation and roundabout production (and consumption). In the final version of Black's theory, there are billions of sectors producing highly differentiated outputs with highly specialized inputs in order to meet highly differentiated final preferences. Further, the flow of output at any moment in time depends on a sequence of inputs stretching back into the past, even the distant past, inputs that potentially continue to contribute to outputs on into the future, possibly even into the distant future. Likewise, the flow of final utility from consuming current output depends on consumption in the past (the utility function is time-inseparable) and current consumption continues to contribute to utility in the future. The current flow of output (and consumption) in any one specific sector is thus linked both cross-sectionally and cross-temporally with the flow of output in every other sector in the past and the future.

From this point of view, economic growth appears as a process of increasing sectoral differentiation and increasing temporal roundaboutness, a process with no apparent end in sight. What we observe as accumulation of capital, physical and human, is just the form that the process takes. The downside of growth by increased specialization is increased vulnerability to technology and taste shocks. "Obsolescence is the dark side of innovation" (1995a, 67).

For Black, and here also he departs fundamentally from Fisher, the central issue is not the balance between aggregate output and aggregate consumption but rather the match between the pattern of output and the pattern of demand. Mismatches happen because investment today depends on a forecast of demand and output patterns in the future, perhaps even the distant future. People do the best they can, but they are bound to make mistakes simply because they cannot see the future very clearly. Even if, at the level of the economy as a whole, the real rate of interest and the price of risk are both constant (which they probably aren't but which Black was inclined to assume as a Bayesian prior) mistakes are inevitable concerning the details. The inevitable mismatch is the fundamental source of risk for individual investments. The volatility of asset prices comes from the fact that the present value of real assets depends on details about the future, about which details we form expectations that shift as our views of the future shift. "Given the volatility of expectations, I'm surprised that asset prices aren't more volatile than they are" (1995a, 59).

Mismatch, or "uncertainty about whether we will have what we want in the future, and about whether we will want what we have" (1995a, 45), is also the fundamental source of risk for the economy as a whole. When the match is good, we have a boom. When the match is bad, we have a recession. Fixing a bad match means shifting the structure of production more into line with the structure of demand, which takes time and resources in an economy where capital is

highly specialized and production very roundabout. By the time we discover that our past expectations were wrong, the investments have already been made, and it takes time for the new investments implied by the new set of expectations to begin producing to meet the new structure of demand. Cyclically persistent unemployment is the result.

Because mismatch risk affects the entire economy, it cannot be diversified away and must be borne by someone somewhere. Mismatch risk is thus the general equilibrium analogue to market risk in the standard CAPM. It follows that financial intermediation can help us to bear mismatch risk by helping us to eliminate other sources of risk through diversification, and by helping to transfer the remaining undiversifiable risk to those most willing and able to bear it. Going further, political institutions such as private property and enforceable contracts can help us to bear mismatch risk by helping us to eliminate still other sources of risk. The important thing for social welfare is not to waste society's limited risk tolerance on forms of risk that offer no compensating reward in higher expected return.

Black viewed the full general equilibrium model as "a single unified theory of almost everything" (1995a, 3). The only really substantial issue on which it does not throw any particularly positive light is the matter of the overall level of prices or the value of the monetary unit, and hence also the matter of the nominal exchange rate that measures the relative value of different monetary units. In Black's general equilibrium, value is measured by an arbitrary unit of account that has no effect on equilibrium. The consequence is that price levels and exchange rates are not determined by the theory and can, in principle, be anything at all. Thus the theory seems to suggest that a country could, relatively easily, peg its absolute price level by intervening in gold markets, or peg its relative price level by intervening in exchange markets. "In modern economies, the inflation rate is indeterminate...It can be whatever people think it will

be...a pure case of self-fulfilling expectations" (1995a, 80). In Black's view, a peg works by giving self-fulfilling expectations an anchor. Fixing the quantity of money has nothing to do with it, and would be inconsistent with equilibrium.

Among the Economists

In following Irving Fisher, Fischer Black was doing nothing more than what economists of all stripes said they were doing at the time he was writing. Irving Fisher's Theory of Interest (1930), a revision that combined his 1906 Capital and Income and his 1907 The Rate of Interest, was a seminal text for post war American economics. Lacking any formal training in economics or finance, Black had every reason to think that by engaging the work of Fisher he would automatically be engaging modern economics. And he had every reason to find Fisher's work a natural entry point into economics since Black, like Fisher, came to economics from mathematics and physics. Irving Fisher was a kindred spirit. Black's choice to enter academia when the opportunity arose can be understood as a choice to follow in Irving Fisher's footsteps.

Redoing Irving Fisher for the case of uncertainty led Black to some rather different conclusions, and it was on precisely those points of divergence that he met resistance, since most economists remained (and remain still) attached to Fisher's original conclusions. First, money. In his 1911 The Purchasing Power of Money, Irving Fisher had revived the quantity theory of money, in part to provide the theory of the price level that was missing from his version of general equilibrium. Post war American economics, both monetarist and Keynesian, followed Fisher's lead, but Fischer Black did not, on the grounds that the quantity theory was inconsistent with equilibrium as he understood it. Ruling out exogenous money left the price level hanging, but there was really no choice since exogenous money was inconsistent with CAPM. Black

made the argument repeatedly and in different forms (1970, 1972c, 1974a, 1978b, 1987) but to little effect, at least among the economists who were his target audience. One reason is that what seemed to him obvious, from the point of view of his CAPM-inspired version of general equilibrium, was by no means obvious from other points of view. Black reasoned from CAPM, but no one else did, nor did they trust conclusions derived from such reasoning, especially conclusions that seemed to revive the dangerous (because inflationary) fallacy of the real bills doctrine.

Second, business cycles. Irving Fisher had analyzed business cycles as disequilibrium phenomena caused by monetary instability (1911 Ch. 4, 1923). In Fisher's theory, nominal interest rates adjust only slowly to changes in the rate of inflation and the resulting changes in the real rate of interest cause real disequilibrium fluctuations. Monetary stabilization that brings price level stabilization can therefore help to stabilize business cycles. Generally speaking, post war economics, both monetarist and Keynesian, followed Fisher's lead--Keynesians emphasized fiscal policy as a more direct stabilization tool--but Fischer Black did not. He thought he could understand business cycles as essentially non-monetary and equilibrium phenomena (1981, 1987) but his argument made little headway with either monetarists or Keynesians. Again, he reasoned from CAPM, but no one else did, nor did they trust conclusions derived from such reasoning, especially conclusions that seemed to revive the dangerous (because quietist) pre war, and pre-Depression, doctrine of laissez faire. Black was willing to make an exception for the Depression, which he analyzed as a disequilibrium caused by a currency trap that kept nominal interest rates positive even as prices were falling (1981, 1995b), but that single exception didn't make his audience much more receptive.

Third, econometrics. Irving Fisher, after completing the main lines of his theoretical edifice (1906, 1907, 1911), turned his attention to application, expansion, and persuasion. Among other activities, he developed the theory of index numbers (1922) and in 1930 founded the Econometric Society. Here again, post war economics, both monetarist and Keynesian, generally followed Fisher's lead, but Fischer Black did not (1982a, 1987 Ch. 12). Because the details matter in Black's theory, a fundamental empirical program would require detailed disaggregated data on technology and preferences, possible in principle but impossible in practice because prohibitively expensive. Further, the two key statistics of his theory, the real rate of interest and the price of risk, are essentially unobservable because we cannot observe market expectations of future inflation or future market returns, so here too an empirical program is unpromising. The closest we can come to observing the two fundamental prices of capital is to observe their effects in specific security prices, so that is where Black focused his efforts. To economists it seemed an unnecessarily narrow empirical program, and one moreover that depended excessively on the dubious assumption of informational efficiency. Unwilling to assume what they felt had yet to be proved, economists did not trust Black's conclusions, especially when they threatened the scientific status of standard econometric practice.

For Black, the intellectual advantages of his conception of equilibrium seemed obvious. Any other conception would necessarily contain the seeds of its own destruction in the sense that there would necessarily be unexploited profit opportunities which, once exploited, would break the equilibrium. This fragility argument was however not so compelling to economists, who were used to thinking of general equilibrium as a long-run reference point only. For economists, the business of economics consists largely in attending to the consequences of the various frictions assumed away in an ideal equilibrium theory, so equilibrium is not a very relevant

model for practical use. For Black, by contrast, a world without frictions seemed a very relevant model of financial markets where prices move instantaneously to reflect new information. By extension, it should also be a relevant model of the larger world since (for him) the significant macroeconomic state variable is the value of wealth as determined in financial markets.

Black's sojourn in academia, starting in 1971 at the University of Chicago and continuing from 1975 at MIT, came to an end in 1984 when he left MIT to join Goldman Sachs. But even after he left academia, he never left the academic debates. The rise in the 1980s of new classical economics, and especially the movement that marched under the banner of "Real Business Cycles" (Long and Plosser 1983) and proposed to treat cycles as equilibrium phenomena, gave Black a new audience, potentially more receptive than the monetarists and Keynesians he was addressing in the 1970s. Significantly, the RBC group rejected the post war neoclassical synthesis and embraced instead the Walrasian general equilibrium model as developed by Arrow and Debreu among others, but not the most general version of that model. Thus, though they rejected Fisher's disequilibrium theory of cycles, they retained Fisher's aggregative emphasis in their preference for representative agent modeling and the standard aggregative neoclassical production function, both of which Black found unnecessarily limiting (1990c). Black's Exploring General Equilibrium (1995) is best understood as an attempt to engage the RBC economists and rally them to his own cause. When Black lists "features of RBC theories that seem arbitrary and unmotivated" (p. 61-63), he is arguing for the advantages of his own extensions to sectoral disaggregation and roundabout production in order to take account of uncertainty in a fundamental way.

Throughout the book, wherever it is possible to translate his theory into the RBC language, Black does so, but it is important to understand those summary formulations in light of

the more complex theory underlying them. For Black, the standard aggregative neoclassical production function is inadequate because it obscures sectoral and temporal detail by attributing current output to current inputs of capital and labor, but he tries anyway to express his views in that framework in order to reach his intended audience. Most important, he accommodates the central idea of mismatch to the production function framework by introducing the idea that the "utilization" of physical capital and the "effort" of human capital can vary over time. This accommodation makes it possible to express his theory in the familiar Cobb-Douglas production function form: $y_t = A_t(e_t h_t)^{\alpha}(f_t k_t)^{1-\alpha}$, where y is output, h and k are human and physical capital, e and f are effort and utilization, and A is a temporary shock (1995, eq. 5.3).

It's familiar math, but the meaning it expresses remains very far from familiar to the trained economist. For one, the labor input has been replaced by human capital so there is no fixed factor. For another, both physical and human capital are measured at market values, and so are supposed to include technological change. This means that the A coefficient is not the usual technology shift factor (the familiar "Solow residual") but only a multiplier, indeed a kind of inverse price earnings ratio, that converts the stock of effective composite capital into a flow of composite output. In effect, and as he recognizes, Black's production function is a reduced form, not a production function at all in the usual sense of a technical relation between inputs and outputs. What Black is after comes clearer when he groups terms and summarizes as $Y_t=AE_tK_t$ (eq. 5.7), where Y is output, E is composite utilization, and K is composite capital. Here the effective capital stock is just a constant multiple of output, and vice versa. It's just an aggregate version of Black's conception of ideal accounting practice (1993c) wherein accountants at the level of the firm seek to report a measure of earnings that can be multiplied by a constant price-earnings ratio to get the value of the firm.

Finance and Economics

Because Black thought of his work as extending Irving Fisher to the case of uncertainty, he thought his work belonged within the main stream of economics. The initial resistance he encountered did not surprise him--new ideas always encounter resistance--but the persistence of resistance in the face of his efforts to overcome it was another matter. In the end he concluded: "I do not fully understand some of the tools and concepts used by those who have had [formal] training" in economics or finance (1995, xi). This seems right, but worth more explication.

In retrospect, the most fundamental source of misunderstanding came (and comes still) from the difference between an economics and a finance vision of the nature of the economy. The classical economists habitually thought of the present as determined by the past. In Adam Smith, capital is an accumulation from the careful saving of past generations, and much of modern economics still retains this old idea of the essential scarcity of capital, and of the consequent virtue attached to parsimony. The financial point of view, by contrast, sees the present as determined by the future, or rather by our ideas about the future. Capital is less a thing than an idea about future income flows discounted back to the present, and the quantity of capital can therefore change without prior saving.

In The Nature of Capital and Income, Irving Fisher (1906) straddled the older world view of economics and the emerging world view of finance by distinguishing physical capital goods (for which the past-determines-present view makes sense) from the value of those goods (for which the future-determines-present view makes sense). By following Fisher, Black wound up employing the same straddle. Even more, because Black focused his attention on the case of

uncertainty, his attention was naturally even more focused on the future-determines-present view than was Fisher's. Uncertainty is all in the future, and certainty is all in the past. As a consequence, though Black followed Fisher, he put even more emphasis on the market value of wealth, and on reasoning in a model without fixed factors, than Fisher ever did. In both respects, Fisher himself had problems getting his theories accepted--his views on the nature of capital and income are still not accepted by economists--and most of Black's problems came from the same source.

A second fundamental source of misunderstanding came (and comes still) from the difference between the liquidity scarcity view of economics and the liquidity abundance view of finance. One of the reasons for the persistent attraction to economists of the quantity theory of money, even given all its faults, is that it expresses succinctly the economist's intuitive sense that the real liquidity of the economy as a whole is scarce and that attempts to increase liquidity by expanding nominal money must eventually reckon with this fundamental real scarcity. The theory of value abstracts from the scarcity of liquidity and treats all commodities as equally and perfectly liquid, but economists recognize that the real world is not like that. This means that monetary theory cannot be brought under the theory of value (as currently constructed) but no alternative analytical structure has yet emerged to gain general acceptance. Until it does, the old distinction between money and credit remains as the economist's crude theoretical attempt to grapple with the apparent hierarchy of liquidity, with credit viewed as a mechanism for stretching scarce liquidity (money). Similarly, the old attempt to measure the quantity of money remains as the economist's crude empirical attempt to grapple with the same hierarchy by measuring the size of its base.

None of this comes through in finance. What seems to an economist as a qualitative difference between more and less liquid commodities, seems to a finance person just a matter of price. The contraction of the liquidity hierarchy that an economist calls a liquidity crisis appears to a finance person as a price discontinuity or a volatility jump. Similarly, system-wide liquidity fluctuations are absorbed by finance people into the more general category of market risk where they disappear as distinct phenomena. In this sense, both Irving Fisher and Fischer Black were finance people at heart. Irving Fisher had no sense of liquidity scarcity, despite his advocacy for the quantity theory, and neither did Fischer Black, despite his rejection of it. Both saw money in equilibrium as essentially just a unit of measurement. For both, equilibrium is a state in which all forms of wealth are equally and fully liquid, so that exchange is fundamentally just a swap of one form of wealth for another, and there is nothing for money to do except provide a unit of account. Summers (1985) has commented on the development of parallel cultures of economics and finance, each addressing the same problems but with widely differing methods. He has furthermore deplored this development, albeit without explaining its origin, as a failure to exploit potential gains from more free intellectual commerce. Black's attempt, throughout his career, to engage in the intellectual commerce recommended by Summers suggests that the obstacles to trade are large and fundamental. Black agreed with Summers that finance should be a subfield of economics, but he could not accept the intellectual constraint that would have come from placing finance under the economics of his own day. Economics had to change first. Black's strategy was to consolidate an intellectual base in finance departments at business schools, and to use that base to promote changes in economics with the long range goal of unifying finance with a changed economics.

Learning from Experience

Finance has yet to revolutionize economic theory in the way Black intended, but it has gone a long way toward revolutionizing the world that economics tries to understand, and here too Fischer Black played a central role. From the very beginning, Black saw in CAPM a vision of the future for the financial services industry. CAPM immediately suggests a natural role for mutual funds that hold the market portfolio and issue shares to individual investors. It also suggests a natural role for banks that facilitate riskless borrowing and lending among households and so allow easy rebalancing as the market fluctuates. Black recognized that the real world of the 1970s was not very similar to the ideal world suggested by CAPM. Why not? There were some good reasons, such as costly information, costly management, and costly selling (Black 1985), all frictions that will cause the real world to deviate systematically from the frictionless ideal even in equilibrium. But mostly the reasons for deviation were historical accident, ill-considered regulation, and plain lack of knowledge of finance, or so Black thought anyway, and these were not good reasons.

Almost all of Black's wide-ranging work in finance falls into place when it is viewed as part of a sustained campaign to overcome the forces of accident and ignorance by spreading the knowledge of finance. One line of work follows from "The Investment Policy Spectrum" (1976c) and leads through Black's effort to spread the knowledge of finance among corporate pension funds, then the largest existing institutional investors (Black 1980a, 1982b, 1989c, 1995e, Black and Dewhurst 1981). The main line of Black's efforts however were directed toward those in the position and with the inclination to set up new institutions to exploit the profitable opportunities afforded by deviations from CAPM. To the extent that CAPM-style mutual funds do not exist, there is a market opportunity for someone (Black 1971a, 1974b,

Black-Scholes 1974b). To the extent that CAPM-style banks do not exist, there is a market opportunity there too (Black 1970, 1975b, Black-Miller-Posner 1978). In both cases, deviation from CAPM means a potentially unexploited opportunity for profit, and exploiting that opportunity means making the world more like the CAPM ideal.

Here again comparison with Irving Fisher proves illuminating. Fisher was a crusader for causes: personal hygiene, prohibition, eugenics, as well as stable money. In each case, he sought to make the world a better place by disseminating the best available scientific knowledge so that the general populace could understand it and act on it. Fischer Black's strategy for social betterment through financial innovation suited his very different character and value system. Unlike Fisher, Black was no crusader, and he had none of the moralist's interest in telling other people how to live. While Black certainly thought there was a right (rational) way to do things and a wrong (irrational) way, unlike Fisher he was quite content to point out what seemed to him the right way and then let people make their own choices.

Black believed that people learn from experience. You don't get them to change by pointing out their sinful ways or even by dangling a vision of CAPM-heaven in front of them. You get them to change by helping them to learn what lessons their own experience has to teach, and by encouraging them to have new experiences in order to learn new lessons. The ultimate result, Black was confident, would be evolution toward the CAPM ideal because entrepreneurs (unlike economists?) would be quick to pick up potentially profitable new ideas and act on them. This explains the enormous importance Black attached to developing simple and workable models. The point was first of all to get a model that people wanted to use, and then let the use itself change consciousness and determine the direction of future evolution. In this way, theory and practice would evolve together.

CAPM was just such a simple and workable model. Black set himself the task of using the model to learn from the experience of historical asset returns. He soon found that historical returns deviate systematically from CAPM in that low beta stocks have done better and high beta stocks have done worse than the theory predicts (Black-Jensen-Scholes 1972). Some said (and still do say) that this historical pattern is evidence against CAPM, but not Black. Instead he interpreted the result as evidence of systematic mispricing arising from borrowing constraints. Aggressive investors are unable to get the exposure they want by holding the market portfolio with leverage, so they are forced instead to bid up the price of high beta stocks as a substitute (Black 1972a, 1993a, 1993b). To the extent this is true, it suggests a profitable opportunity for someone able to overcome the borrowing constraints and put together a leveraged portfolio of low beta stocks. Together with Myron Scholes, Black worked with Wells Fargo Bank to set up such a fund, but it never got off the ground. In Black's mind, that failure only confirmed the theory by confirming that the constraints on which the theory relies are significant in the real world (Black-Scholes 1974b). The profitable opportunity remained and would, Black thought, eventually attract a more successful attempt.

For Black, discovery of the "beta factor" meant not that the one-factor CAPM had to be rejected but that a simple and workable model of security pricing would have to be at least a two-factor model, at least until such a day that borrowing constraints were overcome. Until then, the question was whether two factors are enough, or do we need three? One idea for a third factor was dividends, but Black could find no evidence for it (Black-Scholes 1974a). Instead of a dividend factor, there was a dividend puzzle (Black 1976b, 1990e): Why do firms pay dividends at all?

A more promising potential third factor was domestic bias. In an ideal international CAPM world everyone would hold the world market portfolio. To the extent they are prevented from doing so, they bear more risk for a given expected return which means that a profitable opportunity exists for anyone able to overcome domestic bias (Black 1974c). The failure of Wells Fargo successfully to exploit the second factor however seems to have made Black reluctant to encourage any immediate attempt to exploit the third factor (but see 1978a). Only much later when he was at Goldman Sachs did he find the institutional capability to exploit the opportunity successfully (Black 1992, Black-Litterman 1991, 1992). Even then, adaptation of the international CAPM to practical use required another theoretical innovation. Because clients tended to measure returns in domestic currency, it was necessary to augment CAPM with a model of optimal currency hedging (1989e, 1990a, 1990d).

Because of the importance of the fixed income business to Goldman Sachs, Black also had the opportunity there to follow up on some of his early ideas about bond pricing (for example, FN 14 "The Term Structure of Interest Rates" June 2, 1970). Since bonds have little correlation with stocks, their betas are near zero and CAPM says that expected returns over any holding period should be near the riskless rate for that period. This implication of the theory does not appear to fit the historical facts, but it's hard to know why not without an operational model of equilibrium bond pricing. (Early on, Black suspected that central bank intervention was part of the story, but even so one needs a model of equilibrium pricing in order reliably to detect the second factor.) Black's efforts to build such a model (Black-Derman-Toy 1990, Black-Karasinski 1991, Black-Derman-Toy-Francis 1994) can be understood as analogous to his earlier effort with Jensen and Scholes that tested CAPM and confirmed the existence of the second beta factor. The first step was to push as far as possible with a one-factor model, using

unobserved expected future interest rates and volatilities as free variables in an attempt to fit the observed term structure. Experience with that model would then show if a second factor was really needed. Black became convinced that it was but did not live to complete the necessary work.

Learning about Noise

When Black used CAPM as the foundation of his macroeconomic theory, or as the basis of his vision of the future for financial services, he typically assumed strong form efficiency, in the sense of Fama (1970), and proceeded to work out the logical consequences. There is however another line of work, running parallel, that is concerned with the theoretical and practical foundations of strong form efficiency. From the very beginning to the very end of his work, from "Toward a Fully Automated Stock Exchange" (1971b) to "Equilibrium Exchanges" (1995c), Black explored the problem of how it happens that diffuse private information gets summarized and published in market prices, and also how that process might be made to work even better.

Irving Fisher had grappled with essentially the same problem when he considered the connection between the prices implied by his theory and the actual market prices out in the world. Because he thought the real world was often in disequilibrium, Fisher took the view that his theory of equilibrium prices was a theory of appropriate "appraisal" prices (1906, 11-12), a theory of what prices should be, tended to be, and would be in equilibrium rather than what they actually were. Fischer Black seems to have adopted much the same view, despite his much stronger belief in equilibrium as a positive description of the world. In his view, market prices tend to track appraisal prices pretty well because the people doing the appraisal trade on the

results, but he still found it useful to distinguish appraisal price (or value) from market price (or price).

In his early work, Black seems to have had in mind a possible future ideal world in which almost all trading takes place at appraisal prices between large institutional investors, and for the purpose of raising cash not beating the market (1985). That would be a world with very little trading, which is to say a world very different from the actual world in which there is considerable trading by uninformed individuals ("fools and gamblers") who would be better off buying and holding the market portfolio (1977). It would also be a world in which actual trading could be largely automated and would not have to rely on speculators (specialists and market makers) (1971b). It would be a world in which market prices would be identically equal to appraisal prices because the only trading would be by people who use objective and professional methods to arrive at appraisal values.

The key to the success of such a world, so Black thought at first, is the professional security analyst who combs through available data in search of securities that are worth more or less than their market price. Analysts do not generally beat the market, but in trying to do so they make market prices more efficient. Anything we can do to help the analyst therefore will help to make markets more efficient. The success of Value Line (Black 1973) showed him what a truly professional approach can achieve, and set a standard for others to beat. In this spirit, Treynor-Black (1972, 1973) sketches a method for rationalizing security analysis, a method subsequently reprised in Black-Litterman (1991, 1992) for the international CAPM. The idea is optimally to combine CAPM with independent analysis, allowing each to do what it does best (see also 1988c).

One of the main problems the analyst faces is lack of reliable accounting data (1976d) and this limits his effectiveness. One way around that problem, Black came to see, is to mobilize the accountants. Accountants cannot report the proprietary data needed by analysts without giving away valuable business secrets, but they can use that proprietary data to calculate their own summary appraisal value for the firm, and then report the summary value. In fact, the correlation between reported earnings and subsequent market price suggests that accountants are already doing something like that, but without being fully aware of what they are doing (1980b). (The "magic in earnings" is thus one reason for the success of the Value Line rankings.) Black (1993c) sketches new rules of accounting that, were they adopted, would make the goal explicit. The point is to get both the analyst outside the firm and the accountant inside the firm working toward the same goal of finding the correct market value for the firm. One way of understanding the origin of informational efficiency is that these two professional appraisal values provide the centers of gravity that keep market prices efficient.

The idea that there are objective and professional methods for arriving at appraisal values leaves out, and to some extent opposes, the idea of liquid markets. Black never gave up on the appraisal price ideal but gradually he did begin to think about a less ideal but more realistic vision of the future, one that has a place for speculative trading activity and liquid markets even in equilibrium. The inflection point in his thinking about the problem is "Noise" (1986), Black's presidential address to the American Finance Association in which he shocked his audience by, in effect, renouncing his belief in strong form efficiency in favor of the substantially weaker view that "price is within a factor of 2 of value, i.e. the price is more than half of value and less than twice value" (p. 533). Appraisal values are still the center of gravity, but now the pull of gravity is sufficient only to keep price within the general vicinity. Information traders are

insufficiently strong to impose appraisal values, because it is impossible clearly to distinguish information from noise, and information trading from noise trading.

One consequence is that even ideal equilibrium is not necessarily efficient, though we can always keep trying to do better. One way is to professionalize the production of appraisal values, as mentioned above, with the objective of extracting as much information as possible from available signals. Another way is to devise exchange mechanisms that do a better job of distinguishing information trading which should move the price from other trading which should not. In his early work, Black thought that automation was part of the answer (1971b) but he changed his mind about that (1989d) and decided instead that the key was to introduce a new kind of order. Instead of the existing market orders and limit orders, Black proposed a world of indexed limit orders (1971b) at different levels of urgency (1995c). The point is to force the news trader to partly reveal his intention by choosing a high level of urgency, and to enable other traders to partly protect themselves from news traders by choosing a low level of urgency.

Black died soon after finishing "Equilibrium Exchanges" (1995c) and so never had an opportunity to revise his other theoretical positions in light of his new view that a fully efficient equilibrium was impossible. Nevertheless, he must have recognized that, in allowing for urgency to have a price, he was moving a step away from the liquidity abundance view of finance and toward the liquidity scarcity view of economics. It is perhaps also significant that, at the same time, he was concluding that a one-factor model could not adequately capture what was happening in the term structure of interest rates. After working all his life to get economics to accommodate to finance, was he now beginning to see the need for accommodation in the opposite direction? We'll never know.

Learning About Options

So far nothing has been said about options, nor has anything needed to be said. The world of CAPM is, at root, a world of equity and debt only. Even the debt is short-term debt only, so even bonds and other fixed income securities have no essential place in a CAPM world, much less options. It is for this reason that the success of options markets, subsequent to the 1973 opening of the Chicago Board Options Exchange, surprised and puzzled Black. The success of options became something to explain, and here is a final thread that stretches from the beginning of Black's work to the end, from "The Pricing of Options and Corporate Liabilities" (Black-Scholes 1973, [FN 16, 1970]) to "Hedging, Speculation, and Systemic Risk" (1995d). The evolution of Black's thinking on this question runs parallel with the evolution of his thinking about informational efficiency.

Nowadays, we are so accustomed to no-arbitrage arguments that it takes an effort of will to view the options pricing problem as Black did, as a problem of appraisal pricing. Given the imperfection of markets at the time, the dynamic hedging pricing strategy was of no practical importance, however elegant it might be from a mathematical standpoint. Black's pricing strategy was therefore simply to apply the ideas of CAPM by calculating and pricing the exposure to market risk in an option (Black 1989a). At the time, he never imagined that there would be an active options market, nor was he particularly in favor of one. Commenting on the proposal to establish the Chicago Board Options Exchange, Black wrote: "Options are an exciting way to gamble, and the Chicago Board Options Exchange wants to act as the gambling house and take its cut. There's nothing wrong with that; but if we are to permit this form of

gambling, it seems logical to tax it heavily, as the government taxes betting on horse races."[3] He didn't get his way, options markets became a big success, and at the center of it all was the Black-Scholes formula providing appraisal values for making markets in the new contracts.

Faced with the unexpected success of the formula, Black's main concern seems to have been to educate people about the limitations of the formula and to find ways to adjust for unrealistic assumptions (1975a). From the start he recognized that sometimes the model knows more than the market and sometimes the market knows more than the model (Black-Scholes 1972). If there is going to be a market, better that the market be as efficient as possible, which requires the best possible appraisal prices. (A similar idea seems to lie behind his work to develop a better formula for pricing commodity contracts, Black 1976a.).

Since the options appraisal price is most sensitive to the estimate of future price volatility, Black focused most of his efforts on improving the volatility estimate. Statistical study of stock price volatility (1976e) left him with some few rules of thumb for correcting the formula (1988a, 1989b), but even when he pointed them out, people failed to use them. "Because the formula is so popular, because so many traders and investors use it, options prices tend to fit the model even when they shouldn't" (1990b). The appraisal formula itself had become a source of noise.

One critical moment when the options idea added noise was the crash of 1987 which implicated strategies of portfolio insurance. What people called portfolio insurance was a dynamic hedging strategy designed to work like a protective put option. The problem was that, unlike a put option, portfolio insurance requires trading and so presumes liquid markets.

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[3]The quotation is the conclusion of Black's unpublished paper "A Central Market in Options for Securities: Opportunities and Unrealistic Hopes", n.d. Black Papers, Box 21, Folder "options--exchange--paper".

Furthermore, and again unlike a put, there is no contract with a price to balance supply and demand. Intentions by portfolio insurers to follow a particular trading strategy require, for their success, intentions by someone else to take the other side of the trade, but there is no mechanism to guarantee such a match of intentions. Portfolio insurers may therefore proceed under the assumption that they are insured against market risk without anyone actually undertaking to bear that risk. In Black's analysis, the boom and subsequent crash were caused by the development and discovery of just such incompatible beliefs (1988d). Mean reversion in expected returns is the factor that brings the market for portfolio insurance into equilibrium, but biased beliefs about the degree of mean reversion can keep the market from reaching equilibrium for a while, until the beliefs are corrected.

The general idea of portfolio insurance was present even in Black's earliest work, though he didn't call it that (1988b [FN 6, 1969]; compare Black and Perold 1992). Under certain rather general conditions on the utility function, individuals optimally keep a constant proportion of their wealth in stocks. Aggressive investors use leverage, and a constant leverage ratio means they buy more stocks when the market rises and sell when the market falls. Well before the crash of 1987, Black developed this idea into a formal dynamic trading strategy he called constant proportion portfolio insurance (CPPI) and proposed it as a simpler alternative to the synthetic put option (SPO) portfolio insurance being proposed by others (Black-Jones, 1987, 1988, Black-Rouhani 1989). Recognizing that successful CPPI requires matching a buyer of insurance with a seller, Black also promoted the advantages to the seller, the timid investor who holds a constant fraction of wealth in stocks and so winds up buying as markets fall and selling as they rise (Black-Hakanoglu 1989). Black's idea was that there should be a clearing house to match buyers and sellers. Such a clearing house would ensure that dynamic trading could take

place as planned, and that shifts in relative demand and supply of portfolio insurance would be revealed in the market.

Since CPPI is like a perpetual American call (Black and Perold 1992), the demand for portfolio insurance provides one possible reason for options trading, but still it can't explain the trading we observe in options with finite exercise dates. The volume of options trading therefore remained still a puzzle. "I continue to be amazed at the success of option products" (1990b).

Why are there options? Black's final answer was that the market in options arises as a way of transferring the risk embedded in standard financial contracts such as fixed rate callable mortgage debt (1995d), and presumably also corporate securities that include option-like features (Black and Cox 1976). For whatever reason, people apparently want to speculate on the future of interest rates by issuing such debt, which means that someone has to take the other side of the bet. What options do is to break down the components of the speculation so that each bit can be sold separately, presumably to whoever is most prepared to bear the risk. What options do is to make more apparent how many of the risks we face are man-made.

For Black, learning about options was learning about the extent of irrationality still remaining in human preferences. People remain fools and gamblers to quite a large extent, not yet the expected utility maximizers of our theoretical constructs. Maybe someday they will be more rational, and if so it will be because they have learned to be so. Indeed, since people learn from experience, the best way of teaching people how to be expected utility maximizers may be to allow them to experience the costs of less rational preferences. Fools and gamblers will learn by losing money to those who have already learned the discipline to resist more primitive urges.

Conclusion

"The first step is for the investor to convince himself that the strong form of the random walk hypothesis is true. And this is very difficult for most investors to do" (Black 1971a, 22). One thing that made Fischer Black different from other people is that he did not find it difficult to convince himself of strong form efficiency. It was no more difficult for him to "live up to the model" of expected utility maximization. It suited him to live by these rules. Having convinced himself, he spun a vision of what the world would look like when, as eventually he thought would happen, everyone became convinced of the same lessons. His general equilibrium CAPM is the model of a world in which everyone is as he chose to be.

In 1965, Black learned about CAPM from Jack Treynor. Over the next thirty years he took part in the rise of modern finance that made the most significant elements of the CAPM vision into reality: the automation of trading, the dominance of the institutional investor, the rise of the mutual fund, the deregulation and transformation of banking, the integration and globalization of markets. As the world evolved, elements emerged that Black did not expect--most significant, the success of options and other derivatives--but in time he came to see them as necessary and even helpful during the transition toward the CAPM ideal. As a young man, so strong was the ideal in Black's mind that he thought its arrival was imminent, if not already implicit in the real world. As he aged, he became more and more aware of the persistence of deviations from the ideal, and his forecast date for the arrival of the full CAPM ideal retreated ever farther into the future. Black died recognizing that full equilibrium was yet quite a distance away, but believing it to be still worthwhile to work toward its eventual arrival.

The one disappointment he felt, and it was a big one, had to do with the lack of acceptance of his economic theories. He knew he would be remembered for his contributions to finance, most significantly (if rather ironically) for the options formula,

but he wanted to be remembered for his contributions to economics. His economics is, after all, the theory of the ideal CAPM world that he wanted to see and thought he might live to see. The future relevance of Black's economics will depend on whether future developments in the real world bring us closer to the CAPM ideal or take us farther away. How far is it possible for mere ideas about the future to determine the present unconstrained by conditions inherited from the past? How far is it possible for ever more efficient markets to stretch available liquidity and so bring about the effective abundance assumed by the theory? We live in an age when the finance vision of the nature of the economy is being tested as never before. We'll learn from the experience.

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